Stage 2	Unit 1: Pattern Sniffing	
Stage 1 support overview	**Stage 2 core learning overview**	**Stage 3 extension overview**
➢ count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number ➢ count, read and write numbers to 100 in numerals; count in multiples of twos, fives and tens	➢ count in steps of 2, 3, and 5 from 0, and in tens from any number, forward and backward ➢ order and arrange combinations of mathematical objects in patterns and sequences ➢ recognise odd and even numbers ➢ recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables	➢ **count from 0 in multiples of 4, 8, 50 and 100;** ➢ **find 10 or 100 more or less than a given number** ➢ **recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables**

Key learning steps	Key Vocabulary
1. I can count in steps of 2s, and 5s from 0. I can talk about the number patterns. 2. I can count in steps of 3s from 0. 3. I can count forwards and backwards in 10s from any given number. 4. I can recognise odd numbers. 5. I can recognise even numbers. 6. I can use numicon to make mathematical patterns. 7. I can use mathematical resources to generate sequences. 8. I can recall multiplication and division facts for 2s, 5s and 10s.	Odd Even Sequence Pattern counters cubes blocks rods before after multiplication division

Show me... , And another ...	Convince me	What's the same? What's different? (Odd one out)	Always, sometimes, never
...an odd number bigger than 20 ... an even number between 40 and 50 ...what comes next 25, 30, 35, ...what comes next 20, 30, 40, ...what comes next 42, 52, 62, ... how we count in 10s on our hundred square ... a pattern using counting bears	...that 31 is ten more than 21 ... That 55 comes next 35, 40, 45, 50, ... That 53 is an odd number ... That this is a pattern ... That the answer to 4 x 5 is an even number	1,3,5,8 22,24,26,27 25,30,36,40 33,36,39,41,45 What is the same about 2,12,24,36? What is the odd one out, 3, 10, 13, 23?	If I find 10 more than a number, it will end with the same digit as the number I started with Even numbers always end in 0,2,4,6,8 Even numbers can always be shared. Odd numbers always end in 1,3,5,7,9

...what comes next in the pattern (blocks/numicon/other equipment) ... how you can arrange the numicon pieces into a pattern ... the answer to 3 x 5 etc.			If I add 2 odd numbers the answer is always odd. If I add 2 even numbers the answer is always even. If I add one even number and one odd number the answer will be even.

Misconceptions	**Guidance**
Pupils think that any number with a 3 in it will be odd e.g. 34 When counting in 10s, children forget what to do after 'ninety-something' as they bridge 100 Pupils change the rule in a pattern so that it doesn't flow throughout - they also interpret a pattern as 'pretty' rather than a sequence with a rule....	Pupils need to know why a number is odd and why a number is even - this is a defined property. Use a visual representation like numicon to help 'see' the difference between an odd and even number. The pupil needs to be able to verbalise why numbers are defined as odd/even. Use a counting stick to help children learn their times tables (multiplication and division facts) as well as to start to see how they relate to each other.
Activities	**Show me what you know**
"Count in steps of 2, 3, and 5 from 0, and in tens from any number, forward or backward NRICH: Buzzy Bee *" "Odd and Even Numbers NRICH: How Odd ** NRICH: Even and Odd *" "Order and arrange combinations of mathematical objects in patterns and sequences NRICH: Poly Plug Pattern * NRICH: Triple Cubes * NRICH: A City of Towers ** NRICH: Caterpillars **"	Click here to access files in Google drive

Stage 2	Unit 2: Investigating Number Systems	
Stage 1 support overview	**Stage 2 core learning overview**	**Stage 3 extension overview**
➢ read and write numbers from 1 to 20 in numerals and words. ➢ identify and represent numbers using objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most, least	➢ read and write numbers to at least 100 in numerals and in words ➢ recognise the place value of each digit in a two-digit number (tens, ones) ➢ identify, represent and estimate numbers using different representations, including the number line ➢ use place value and number facts to solve problems ➢ compare and order numbers from 0 up to 100; use <, > and = signs	➢ **read and write numbers up to 1000 in numerals and in words** ➢ **recognise the place value of each digit in a three-digit number (hundreds, tens, ones)** ➢ **identify, represent and estimate numbers using different representations** ➢ **solve number problems and practical problems involving these ideas** ➢ **compare and order numbers up to 1000**

Key learning steps	Key Vocabulary	
1. I can read and write numerals and words from 10 to 50. 2. I can read and write numerals and words from 50 to 100. 3. I can say the number that is one more or less than a given number to 100. 4. I can understand the value of each digit in a 2-digit number and partition the number as tens and ones 5. I can represent any number up to 20 using: - unifix cubes oe - a number line - a hundred square - numicon - cuisenaire rods" 6. I can solve problems using my knowledge of number facts and place value. 7. I can compare up to three numbers or amounts up to 100 and say which is the most and which is the least (or which is more or less if only 2 numbers). 8. I can compare two numbers up 100 and use the signs <, > (and =) to show this comparison.	tens hundreds one/two/three digit numbers place, place value stands for / represents place holder one hundred AND *****	compare position greater than / less than order =, > <

Show me... , And another ...	Convince me	What's the same? What's different? (Odd one out)	Always, sometimes, never
... the number forty-two in symbols ... the number 63 in words	... that 88 comes after 87 ... that 23 is represented by 2 tens and	65, 56, 6, 5, 60, 50 1, 10, 100	There is only one possible answer to this gap question ..... > 99

... how we can represent the number 35 using - unifix cubes - the beadstring - the dienes rods - the cuisenaire rods - the numicon - the hundred square - the number line ...where 62 would be on this blank paper strip that goes from 0-100? ...which is a greater 39 or 93? ... one more than 65 ... a number that could complete 54 < ....a number in between 45 and 76	3 ones/this apparatus ... that 38 < 83	eighty, eighty-one, eighteen 49, 50, 51 41>32, 76< 85, 50 = 50, 54<45	If you take a number and reverse its digits, you get a number that is bigger than you started with

Misconceptions	Guidance
Children confuse the order of tens and ones and so will think that fifty-eight can be represented as 85 Children still muddle tens and teens e.g. they will read 80 as eighteen Children think of every number separately e.g. 23 is 23 ones and they cannot see it also as 2 tens and three ones Children get confused about 'which way round' the < and > signs go Children interpret = as ' the answer is' rather than 'is worth the same as' - make	You need to develop a strong understanding of place value so that children see that they only need to actually learn the numbers 0-9 because all other numbers just use these. Try representing numbers using the following place value progression to secure the understanding: - objects with 1:1 correspondence e.g.unifix cubes or straws which can be grouped into tens and 'bundled' but therefore also 'unbundled' - objects that show tens but cannot be moved e.g. dienes rods, cusinenaire rods, numicon or base 10 - objects that represent tens but don't look like tens e.g. place value counters

sure you model this carefully and avoid implying that it simply starts the answer!	or double-sided counters where one colour is tens and the other is ones Work both ways i.e. representing a given number with equipment but also reading the number from the equipment. Also encourage children to find other representations of the same number. Remember - the number line is more abstract than either a beadstring or a hundred-square so progress towards this gradually.
Activities	**Show me what you know**
NRICH: Snail One Hundred * NRICH: How We'd Count * NRICH: Tug of War * NRICH: Sort Them Out (1) * NRICH: Domino Sequences * NRICH: Domino Number Patterns ** NRICH: Next Domino * NRICH: 100 Square Jigsaw * NRICH: That Number Square! * NRICH: I Like … * NRICH: Light the Lights *** NRICH: Largest Even *	Click here to access files in Google drive

Stage 2	Unit 3: Solving Calculation Problems	
Stage 1 support overview	**Stage 2 core learning overview**	**Stage 3 extension overview**
➢ represent and use number bonds and related subtraction facts within 20 ➢ add and subtract one-digit and two-digit numbers to 20, including zero ➢ read, write and interpret mathematical statements involving addition (+), subtraction (–) and equals (=) signs	➢ recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 ➢ "add and subtract numbers using concrete objects, pictorial representations, and mentally, including: - a two-digit number and ones - a two-digit number and tens - two two-digit numbers - adding three one-digit numbers" ➢ show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot	➢ **"add and subtract numbers mentally, including:** **- a three-digit number and ones** **- a three-digit number and tens** **- a three-digit number and hundreds"** ➢ **add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction** ➢ **estimate the answer to a calculation and use inverse operations to check answers**

Key learning steps	Key Vocabulary	
1. I can quickly recall addition and subtraction facts to 20 and use them to derive facts to 100 2. I can mentally add and subtract ones to and from a two-digit number 3. I can add and subtract tens to and from a two-digit number 4. I can add and subtract two two-digit numbers that do not bridge ten with equipment. 5. I can add any two two-digit numbers (using apparatus and recording) 6. I can subtract any two two-digit numbers (using apparatus and recording) 7. I can add three one digit numbers 8. I can say whether the order matters in a statement	add subtract ones tens hundreds digit more than less than sum total altogether one more	one less take away exchange column order true/false dienes blocks numicon place value grid place value counters value (of a counter or block)

Show me... , And another ...	Convince me	What's the same? What's different? (Odd one out)	Always, sometimes, never
... two numbers that are easy to add ... two numbers that are hard to add ... two numbers that are easy to subtract	... that the number 60-12 is another way of saying 72 ... that if I add ten to a number I will get	30, 70, 20, 80 60+40, 50+50, 30+80, 10+90 90-50, 80-40, 100-60, 70-40	A double digit number add a single digit number equals a double digit number A double digit number add a double

... two numbers that are hard to subtract ... two numbers with a sum of 20 ... two numbers with a sum of 17 ... two numbers with a sum of 50 ... two numbers with a difference of 20 ... three one digit numbers with a sum of 15	the same digit in my 1s column as a I started with ... that 180 - 40 = 140 ... that the answer to 45 + 28 will be the same as the answer to 28 + 45 ... that the answer to 7 + 9 + 6 will be the same as 6 + 9 + 7 AND 9 + 7 + 6. ... that there are 5 more sums with the same numbers that give the same answer as 4 + 6 + 8		digit number equals a double digit number A double digit number subtract a double digit number equals a double digit number Addition makes a number larger Subtraction makes a number smaller

Misconceptions	**Guidance**
Children struggle to add two digit numbers when their place value understanding is weak. If they do not read a number like '52' as 5 tens and 2 ones then they struggle to combine the ones and the tens from two numbers appropriately. When subtracting, children will sometimes subtract the larger number from the smaller initially. They may also fail to exchange a ten for 10 ones when necessary and simply find the difference between the digits in the column e.g. 43 - 27 - they may find the difference between the 3 and the 7 in the ones column as 4 rather than exchanging a ten for 10 ones to get 13 - 7. When exchanging does happen, children sometimes forget to 'remove' the 10 and so end up with ten too many in the answer.	The aim of this unit for these children is to explore how we can more efficiently add up two numbers beyond the known methods of counting all or counting on. Therefore, all the techniques at Stage 2 rely on some understanding of place value and so this must be secure first. You should make reference to the calculation policy specifically when teaching this unit as the range of equipment and the language are modelled there. It is advised that teachers enable children to practice adding 2-digit numbers practically first (initially with equipment where the 'ten-ness' of a ten is obvious e.g. numicon or dienes). Begin with examples where you add 1s only, then 10s only then a combination of both that does not bridge ten. This will build up children's skills in representing numbers with the apparatus, combining the numbers and then 'reading' the solution. When learning about addition that bridges 10, children need to literally and practically exchange 10 ones for a ten so that they can see how this happens when we come to the written method. More advanced children can carry out this procedure with apparatus such as place value counters where the value of the counter is not obvious from its appearance. Similarly, when learning about subtraction that bridges 10, children need to physically exchange a ten for 10 ones to enable them to carry out the

	subtraction of the 1s. They can eventually move on to doing this with more abstract equipment such as place value counters. To begin to embed the written routines of the calculation policy, it is advised that children work in pairs with one child manipulating the equipment and saying what they are doing aloud while the other child records the calculation using the column method so that they learn that the column method is just a written representation of the practical process (rather than a 'different' method). Lastly, it is important to let children explore whether order matters when we add and subtract - this is a great opportunity for some early investigation. Does 44 + 27 give you the same answer as 27 + 44? etc.
Activities	**Show me what you know**
NRICH: Number Round Up *** NRICH: 4 Dom *** NRICH: Strike it Out * NRICH: Cuisenaire Environment * NRICH: Jumping Squares ** NRICH: Number Balance **	Click here to access files in Google drive

Stage 2	Unit 4: Exploring Shape	
Stage 1 support overview	**Stage 2 core learning overview**	**Stage 3 extension overview**
➢ "recognise and name common 2-D and ➢ 3-D shapes, including: - 2-D shapes [for example, rectangles (including squares), circles and triangles] - 3-D shapes [for example, cuboids (including cubes), pyramids and spheres]"	➢ "identify and describe the properties of ➢ 2-D shapes, including the number of sides and line symmetry in a vertical line" ➢ "identify and describe the properties of ➢ 3-D shapes, including the number of edges, vertices and faces" ➢ compare and sort common 2-D and 3-D shapes and everyday objects	➢ **identify horizontal and vertical lines and pairs of perpendicular and parallel lines** ➢ **recognise angles as a property of shape or a description of a turn** ➢ **identify right angles, recognise that two right angles make a half-turn, three make three quarters of a turn and four a complete turn; identify whether angles are greater than or less than a right angle**

Key learning steps	Key Vocabulary	
1. I can identify the number of sides on a 2d shape 2. I can identify the number of vertices on 2D shapes 3. I can recognise if a 2D shape has a vertical line of symmetry and say where this is 4. I can identify the number of faces on a 3D shape. 5. I can identify the number of edges on a 3D shape 6. I can identify the number of vertices on a 3D shape 7. I can sort 2D shapes and objects using one criterion. 8. I can sort 3D shapes and objects using one criterion.	circle square triangle rectangle pentagon hexagon octagon cube cuboid sphere pyramid prism cylinder sort shape	sides corners vertices and vertex edges faces sides symmetry and line of symmetry objects 2D 3D right angles left right criterion

Show me... , And another ...	Convince me	What's the same? What's different? (Odd one out)	Always, sometimes, never
... a shape with three vertices	... that this is a rectangle	circle; sphere; cylinder; cube	Shapes have the same number of sides as vertices

... a shape with four sides ... a shape where all the sides are equal ... a shape with a line of symmetry ... a shape with more than 1 line of symmetry ... a pentagon ... a sphere ... a shape with six faces ... all the shapes with 12 edges ... all the shapes that are 3D ... all the shapes with no vertices .... all the shapes with a right angle	... that this shape will roll ... that this shape has a line of symmetry ... that this shape does not have any lines of symmetry	cube; cuboid; square; sphere triangle, rectangle, pentagon, circle	2D shapes have 1 line of symmetry A square is a type of rectangle

Misconceptions	**Guidance**
"Some children believe that corner and vertex are not interchangeable - try to use both words as these do both apply to 2D and 3D shapes. Children miscount vertices and sides sometimes because they forget where they have started counting. Some children are surprised that there is more than one shape with, for example, four sides and they therefore assume that it must be a square because they know a square has four sides. Children can infer that part of the definition of a shape is its orientation, colour	"Suggestions in stage 1 may still be relevant and adapted. Consider making each child a shape fan (like a number fan!) that they can use for show-me activities on the carpet or for investigating shape features. Play 'guess the shape' or 'I'm thinking of a shape' games where children describe the properties to see if they match those of the shape in question. Encourage a wide range of criteria when sorting - try to use the SAME set of shapes and sort them differently so that children can see that there isn't a single 'right answer' to this. Use different structures also to sort - you can introduce Carroll diagrams and

etc. so ensure these are varied to avoid inadvertently implying a property about circles that is not intended!"	even Venn diagrams to help you. Don't be frightened of introducing a wider range of shapes - many children of this age can handle a significant amount of new vocabulary such as hemisphere or cylinder or octagon. You may even begin to be able to look at types of triangle with the most able to see the difference between an equilateral and the others. You may NEED to do this to show the children some shapes WITHOUT symmetry ..... Be aware that we are ultimately moving to the idea that a rectangle is the family name and the family divides into squares and oblongs - if you can prepare the way for this it will help! Lines of symmetry-Use of folding paper, mirrors, butterfly paintings, can support initial explanations. Symmetry in art, play, photos etc. helps to develop a more secure comprehension. Two lines and more of symmetry can be easier to see when able to physically manipulate the equipment- use of materials, ribbons, rulers, patterns. rugs, ICT games for co-ordinates and colouring squares. Outdoor grids when children have to stand on squares, counting positions- opposite. Take photo of child's face- fold in half children to paint other side- model how to measure features for a more accurate finish. Children can use a see though square as a right angle finder- they can explore all shapes to find right angles as a right angle investigator."
Activities	**Show me what you know**
NRICH: Shapely Lines * NRICH: Chain of Changes ** NRICH: Colouring Triangles ** NRICH: Exploded Squares * NRICH: Complete the Square *** NRICH: Let's Investigate Triangles * NRICH: Poly Plug Rectangles * NRICH: Square It *	Click here to access files in Google drive

NRICH: Inside Triangles *** NRICH: Building with Solid Shapes * NRICH: Skeleton Shapes ** NRICH: Rolling That Cube *	

Stage 2	Unit 5: Generalising Arithmetic	
Stage 1 support overview	**Stage 2 core learning overview**	**Stage 3 extension overview**
➢ represent and use number bonds and related subtraction facts within 20 ➢ add and subtract one-digit and two-digit numbers to 20, including zero ➢ solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = ☐ – 9 ➢ read, write and interpret mathematical statements involving addition (+), subtraction (–) and equals (=) signs	➢ recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 ➢ "add and subtract numbers using concrete objects, pictorial representations, and mentally, including: - a two-digit number and ones - a two-digit number and tens - two two-digit numbers - adding three one-digit numbers" ➢ recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems. ➢ show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot	➢ **"add and subtract numbers mentally, including:** **- a three-digit number and ones** **- a three-digit number and tens** **- a three-digit number and hundreds"** ➢ **add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction** ➢ **solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction**

Key learning steps	Key Vocabulary	
1. I can recall and use addition and subtraction facts of pairs of numbers up to 20. 2. I can recall pairs of numbers up to 10 and 20 and use them to find related facts of pairs of numbers up to 100. 3. I can add and subtract a two digit number with a one digit number using manipulatives, pictorial representations and mentally. 4. I can add and subtract a two digit number with a tens number using manipulatives, pictorial representations and mentally. 5. I can add and subtract a two digit number with a 2 digit number using manipulatives, pictures and mentally. 6. I know that if I add my numbers in any order I will get the same answer. 7. I can add 3 one digit numbers together using manipulatives, pictures or mentally.	inverse opposite subtract subtraction take away leaves add addition	altogether makes largest smallest total fact family

8. I can work out the subtraction facts from addition facts using the inverse operation. 9. I can use the inverse of addition and subtraction to find missing numbers.	

Show me... , And another ...	Convince me	What's the same? What's different? (Odd one out)	Always, sometimes, never
... two numbers that sum to 20 ... two numbers that have a difference of 7 ... three numbers that sum to 100 ... how you can add 19 to a number ... how you can add 34 and 47 together using equipment ... how we can find 76 - 48 using equipment ... the fact family for 13 + 4	... that 13 + 4 has the same answer as 4 + 13 ... that 16 + 23 has the same answer as 16 + 20 + 1 + 1 + 1 ... that 56 - 39 leaves us with 17 ... that there are more than five pairs of numbers that make 20 ... I have 18 pencils. I take away 3 and there are 15 left. ... you need to use addition/subtraction to solve this problem 'there were 19 people on the bus then four got off, how many are left?'	15 + 9; 9 + 15; 24 - 9; 24 - 15. 11 + 8; 12 + 7; 13 + 6; 14 + 5	Addition is commutative i.e. the order doesn't matter Subtraction is commutative i.e. the order doesn't matter

Misconceptions	Guidance
Children confuse the order of their facts e.g. they know that 16 + 5 = 21 and so they say that 21 + 5 = 16 Children do not see the link between number bonds to 10 and number bonds	Children need to become familiar with a range of manipulatives such as dienes, cuisenaire, place value cards, 100 squares, dominoes, dice etc. in order for them to choose the best resource to model their calculation. Children can use manipulatives to explain their thinking, consolidating their

to 100 e.g. if 4 + 6 = 10 then 40 + 60 will make 100 Children do not use place value when adding and subtracting - signs of this can be them counting repeatedly from 0 or failing to use models that group tens differently. They may need to use a wider range of representations to develop this idea more strongly - some children can do this when the model 'looks' like 10 but not when the visual link has gone. Children do not always understand that addition can be done in any order but that subtraction is not commutative.	understanding but also may highlight any misconceptions. Ensure you use a progression through place value to support the embedding of formal methods later - you would expect children to add by grouping into tens initially (using unifix for example) before gradually doing this with bundles of straws, then numicon, then dienes rods and then place value counters/ Focus a great deal of attention on ensuring number facts are embedded and recall is strong and fast - these skills are essential to good calculation later and children need to know their number bonds off by heart. Look at equivalent calculations in detail - if we know that 21 + 7 is 28, what else do we automatically know?
Activities	**Show me what you know**
NRICH: Cuisenaire Environment * NRICH: Jumping Squares ** NRICH: Number Balance ** NRICH: The Add and Take-away Path * NRICH: Secret Number ** NRICH: How Many? * NRICH: What Was in the Box? * NRICH: Doing and Undoing *	Click here to access files in Google drive

Stage 2	Unit 6: Reasoning with Measures	
Stage 1 support overview	**Stage 2 core learning overview**	**Stage 3 extension overview**
➢ recognise and know the value of different denominations of coins and notes	➢ recognise and use symbols for pounds (£) and pence (p); combine amounts to make a particular value ➢ find different combinations of coins that equal the same amounts of money ➢ solve simple problems in a practical context involving addition and subtraction of money of the same unit, including giving change	➢ **add and subtract amounts of money to give change, using both £ and p in practical contexts** ➢ **measure the perimeter of simple 2-D shapes**

Key learning steps	Key Vocabulary	
1. I can make small amounts of money in different ways using 1p and 2p coins and record the result (eg as 7p = 2p + 2p + 2p + 1p) 2. I can make small amounts of money in different ways using 1p, 2p, 5p and 10p coins and record the result (eg as 17p = 10p + 5p + 2p) 3. I can make larger amounts of money in different ways using any coins and record the result using p or £ (eg as £1.50 = £1 + 50p) 4. I can add up and record the total cost of 2 or 3 items in pence up to 99p. 5. I can select appropriate coins to pay for one or more items in a pretend shop 6. I can add up and record the total cost of 2 or 3 items where the total is over £1 . 7. I can use counting on or formal subtraction to work out change. I can select appropriate coins to give change for an item in a pretend shop 8. I can compare the sizes of objects using informal measures like handspans. I realise that not all people have the same handspans and begin to appreciate the need for standard units.	add change compare cost handspan item larger like measure more	record result same shop smaller subtract take-away total

Show me... , And another ...	Convince me	What's the same? What's different? (Odd one out)	Always, sometimes, never
How to give change when I buy things for 12p and 5p and give a 20pcoin	That 20p is not enough money to buy three 7p sweets. That the table will fit through the door (eg by measuring)	The teacher measures her table as 3 handspans high and the pupil measures his table as 3 handspans high, but the teacher's table is taller. Explain.	The shopkeeper will be able to give the right change.

Misconceptions	Guidance
Writing p as 9; lack of arithmetic skills; lacking concept of not having right money and needing change; need for handspans to be same size	A critical learning point is the ability to record, using correct terminology. Children can show creativity in finding all the ways of making eg 11p (11 x 1p, 2p + 9 x 1p, etc) but will probably record using '+' only (eg 5p + 2p + 2p + 2p = 11p). The idea of giving change is not always easy for pupils and may require some work. It can link to subtraction on a number line if the 'jumps' are restricted to coin amounts. The odd lesson on handspans is needed to establish the purpose of measuring and the problems with non-standard units. It prepares the way for later work using cm.
Activities	**Show me what you know**
NRICH: Five Coins ** NRICH: Money Bags ** NRICH: The Puzzling Sweet Shop **	Click here to access files in Google drive

Stage 2	Unit 7: Discovering Equivalence	
Stage 1 support overview	**Stage 2 core learning overview**	**Stage 2 extension overview**
➢ recognise, find and name a half as one of two equal parts of an object, shape or quantity ➢ recognise, find and name a quarter as one of four equal parts of an object, shape or quantity ➢ identify and represent numbers using objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most, least	➢ recognise, find, name and write fractions 1/3, 1/4, 2/4 and 3/4 of a length, shape, set of objects or quantity ➢ write simple fractions for example, 1/2 of 6 = 3 and recognise the equivalence of 2/4 and ½ ➢ identify, represent and estimate numbers using different representations, including the number line	➢ **recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators** ➢ **recognise and use fractions as numbers: unit fractions and non-unit fractions with small denominators** ➢ **compare and order unit fractions, and fractions with the same denominators** ➢ **recognise and show, using diagrams, equivalent fractions with small denominators** ➢ **identify, represent and estimate numbers using different representations**

Key learning steps	Key Vocabulary	
1. I can find a half, third or a quarter of a number of objects or quantity by sharing into 2, 3, 4 equal groups respectively. 2. I can find a half, third or quarter of a number of objects or quantity by grouping into 2s, 3s or 4s respectively. 3. I can show 1/2, 1/3, 1/4 of a shape or length by dividing it into two, three or four equal parts respectively. 4. I can find 2/4 or 3/4 of a number of objects or quantity by first finding 1/4. 5. I can recognise that the fractions 2/4 and 1/2 are equivalent and show this for a given quantity. 6. I can write simple fractions of quantities to show my calculations e.g. 1/2 of 6 =3 to show finding a half of 6 or 1/3 of 15 = 5 to show finding a third of 15. 7. I can represent a third, a quarter and a half using a range of models and images (including the fraction symbols themselves i.e. 1/2, 1/3, 1/4 and the number line). 8. I can represent the quarter family (1/4, 2/4, 3/4) using a range of models and images (including the fraction symbols and the number line).	half third quarter calculate find equivalent quantity estimate compare order equal to	fraction family 1/2 1/4 1/3 2/4 3/4 halve fourth

Show me... , And another ...	Convince me	What's the same? What's different? (Odd one out)	Always, sometimes, never
... 1/2 of 12 ...1/3 of 12	... that two quarters is worth the same as one half	half, halve, 1/2, 2/4	... you can find one half of 19

... 1/4 of 12 ... how you can find 1/3 by sharing ... how you can find 1/3 by grouping ... 1/4 of this shape? 2/4 of this shape? 3/4 of this shape? ... 2/4 of this shape? 1/2 of this shape? What do you notice?	... that sharing and grouping give you the same result ... that one half of 20 sweets is bigger than one half of 10 sweets ...	1/4, 2/4, 3/4, 4/4	... three quarters of an amount is larger than one half of an amount ... you can only find one half of even numbers ... there is no such thing as 4/4

Misconceptions	**Guidance**
Children think that 1/2 is smaller than 1/4 because 2 is less than 4. Children think that you can find a fraction by dividing the shape up into the correct number of pieces, whether or not these are equally sized. For example, to find a third, they may cut a shape into 3 differently sized pieces. Children do not always see the connection between fractions and division - they do not see 1/2 as 1 divided by 2 and they don't understand the connection between these concepts. Children tend to stick to one particular representation of a fraction (often the circular one) and do not recognise other models or images as also being worth the same.	Once again, there is a need to ensure that children can use division to find fractions of quantities - they should be comfortable with BOTH sharing and grouping representations for this. In Stage 2, it is also important to start to relate fractions to arrays as a pictorial representation of both the fraction as well as a way of making the connection with multiplication and division. For example, showing a third of 12 by arranging twelve items into an array with three rows (because we are finding a third) and counting how many columns this creates. Children need to be confident that finding a third of something is the same as dividing by 3 - the array encourages that connection to be made (and will alter be linked to formal written division also). Similarly, this is a good time to introduce the bar model as a way of representing unit fractions (and then non-unit examples). This will reinforce the process of carrying out a division to find a unit fraction - you can create your own bar images at http://www.mathplayground.com/ThinkingBlocks/thinking_blocks_modeling%20_tool.html With all these representations, it is important that children realise the criticality of dividing the quantity or object into EQUAL parts - so emphasise the use of the same size blocks in the bar model. When exploring representations, do not forget the symbols themselves - it is worth

	exploring that 1/2 means 1 divided by 2 and so on. You can see a lovely example of children exploring fractions as a sharing model on YouTube here: https://www.youtube.com/watch?v=Q-yichde66s&feature=youtu.be
Activities	**Show me what you know**
NRICH: A Bowl of Fruit http://nrich.maths.org/218 Fraction Manipulatives - exploring equivalence http://donnayoung.org/math/fraction.htm Fraction models and support questions - http://www.annery-kiln.eu/gaps-misconceptions/all-images.html	Click here to access files in Google drive

Stage 2	Unit 8: Investigating Statistics	
Stage 1 support overview	**Stage 2 core learning overview**	**Stage 3 extension overview**
➢ "measure and begin to record the following: - lengths and heights - mass/weight - capacity and volume - time (hours, minutes, seconds)	➢ interpret and construct simple pictograms, tally charts, block diagrams and simple tables ➢ ask and answer simple questions by counting the number of objects in each category and sorting the categories by quantity ➢ ask and answer questions about totaling and comparing categorical data	➢ **interpret and present data using bar charts, pictograms and tables** ➢ **solve one-step and two-step questions [for example, 'How many more?' and 'How many fewer?'] using information presented in scaled bar charts and pictograms and tables**

Key learning steps	Key Vocabulary	
1. I can list the categories of data for a tally chart 2. I can record results in tally chart 3. I can produce a frequency table from a tally chart 4. I can construct a pictogram with one picture per item from a tally chart 5. I can construct a block diagram/chart from a tally chart 6. I can read results from a table, pictogram or block diagram. 7. I can use my readings to answer simple questions about the data 8. I can use my readings to make comparisons with the data	results data list categories tally (chart) frequency table pictogram key symbol	block diagram/chart construct most least difference total labels axes

Show me... , And another ...	Convince me	What's the same? What's different? (Odd one out)	Always, sometimes, never
... what a tally for 4 looks like.... 5? .... 6? ... 34? ... a list of data with three categories ... the frequency table for this tally chart ... what you would label each axis on my chart ... how you would represent 3 on this	... that this pictogram and this block graph come from the same list of data ... that there total number of pieces of data is 30	table, block graph, pictogram, tally, list	... A block graph shows what people like or don't like ... The best way to collect information is to ask your friends ... A pictogram is the best way to display data

pictogram			
Misconceptions		**Guidance**	
Children may count a piece of data twice or miss some out when completing a tally chart from a list of data - this can be exacerbated if they have collected their own data and recorded it somewhat haphazardly initially. Children sometimes use different symbols for different categories in pictograms, making it hard to compare. They need to use the same symbol consistently throughout the pictogram. Children forget to bundle in 5s when completing a tally chart and may not count in 5s to save time when totaling. Children use limited vocabulary when describing findings e.g. they say that ""category A was the best"" when in fact they mean the most popular or the longest or the heaviest etc. They also avoid using the word 'frequency' and instead use total/amount. Children do not always think about the context of the data when they are reading from it. e.g. they assume that the highest number is the best		This unit focuses on BOTH representing and interpreting data so ensure you divide your time appropriately to give sufficient focus to reading from graphs and using this to answer questions. One approach to this unit is to complete the full data handling cycle to collect, record, represent and then analyse the data. It is crucial to let children see what happens if you do not organise your data or label your axes and so on so that they realise the importance of doing things properly. Try to model correct language e.g. frequency rather than amount or total, represents for the key on pictograms, block diagram/chart rather than bar chart When teaching block charts, it can be useful to have pre-printed blocks that the children colour in rather than constructing from scratch	
Activities		**Show me what you know**	
		Click here to access files in Google drive	

Stage 2	Unit 9:Solving Number Problems	
Stage 1 support overview	**Stage 2 core learning overview**	**Stage 3 extension overview**
➢ solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher	➢ calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs ➢ show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot ➢ solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts	➢ **write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods** ➢ **solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects**

Key learning steps	Key Vocabulary	
1. I can explain what different multiplication and division number statements mean. 2. I can solve multiplication and division problems using practical equipment to create groups. 3. I can solve multiplication and division problems using arrays. 4. I can use the signs x,÷,and = to record a multiplication or division number sentence. 5. I can show that if I multiply my numbers in any order I will get the same answer. 6. I can show that if I divide my numbers in a different order I will get a different answer. 7. I can solve multiplication and division problems by recalling known facts. 8. I can decide which calculation to carry out and pursue this to solve a problem in context.	multiply x times represent product lots of / groups of array row column divide	÷ divided by division divided into share group quotient

Show me... , And another ...	Convince me	What's the same? What's different? (Odd one out)	Always, sometimes, never

... how you can represent 10 x 4 in as many ways as possible ... how you can represent 15 ÷ 5 ... how you would write a multiplication sentence for 'the number that is twice as big as 8' ... the array for 4 x 5 ... a number sentence you can make using 5, 3 and 15.	... that multiplication is commutative ... that division is commutative ... that 12 divided by 3 and 12 divided into 3 have the same answer ... that 2 x 30 = 60 (using what you know about 2 x 3) ... that is 4 x 2 = 8 then 8 ÷ 2 must be 4	5 10ps, 10 5ps, 10 x 5, 5 x 10, 5 lots of 10, 10 lots of 5, 3 x 4 and 4 x 3 12 ÷ 6 and 6 ÷ 12 2 x 6, 12 x 1, 3 x 4, 4 x 2, 6 x 2, 1 x 12	... if you know that a x b = c then you can make another three true number sentences about these numbers ... you get the same answer whichever way round you multiply ... you get a different answer if you divide in the other order

Misconceptions	**Guidance**
Children may assume that, since multiplication is commutative, division is commutative and can be done in any order! Children struggle with the range of language for both multiplication and division - they may incorrectly read 3 x 4 as 3 lots of 4 rather than 3 multiplied by 4 (which is actually therefore 4 lots of 3) and so on. This can lead to errors in representing the array correctly (which we want to avoid to enable them to 'see' commutativity and to later represent more complex multiplication like 13 x 4 in a consistent way). Children may not see how an array can be used to support division, only multiplication. Children tend to use the 'lots of' representation [repeated addition] of multiplication much more than scaling. Similarly, they may use sharing more than grouping (which makes using an array for division harder). Children tend to stick to their favourite representations - they may overly rely on, for example, numicon when a bead string or an array could be more helpful.	In Stage 2, we need to formalise what has been learnt in Stage 1, both in terms of using correct mathematical language and symbols AND being strict about how we represent problems. Therefore, it is important to ensure that children can represent multiplication and division in a range of ways such as: Multiplying: - with objects grouped (to represent repeated addition) - e.g. 3 x 4 would be 4 groups of 3 objects - with a bead string (4 groups of 3 beads) - the colours help them to 'read off' the answer easily - with a number line (builds from the bead string) - with a length that has been doubled, tripled etc. e.g. 3x4 would be a length of 3 that has been quadrupled - with an array (3x4 would be 3 columns across replicated in 4 rows down) Dividing - with objects that are shared out e.g. 18 ÷ 3 would be 18 items counted and then shared into 3 piles (divided into 3)

	and the number in each pile counted - with objects that are grouped e.g. 18 ÷ 3 would be 18 items that are grouped into 3s and the number of groups counted - with a bead string (18 beads grouped into 3s) - with a number line (repeated subtraction) - with a length that has been divided (link to fractions) e.g. 20 ÷ 4 would be a length of 20 divided into 4 equal pieces (quarters) - with an array (20 ÷ 4 would be 20 items arranged into rows of size 4 (i.e. 4 columns) so that you can see how many rows you can make) See the calculation policy for more on this. Once the representations are clear, make sure children can also read the formal symbols as both words and calculations. Arrays are the most challenging, but the most useful(!), of the representations above because they will form the basis of our later work on mult and div. Therefore, spend time getting these right and recording the number sentences alongside the array. As children get more confident they can draw the array rather than making it from objects to speed things up. Eventually some children may only imagine the array - this is when you've cracked it! Try to stick to examples using their times tables (although you can go outside this when you allow counting) Commutativity - get children to explore an array for something like 2 x 6 and compare it with one for 6 x 2 to see what they notice. Watch the video on teaching the commutative law at the NCETM website to see more about this. https://www.ncetm.org.uk/resources/40530 Division is much harder to approach as, when you reverse it you will end up with a smaller number divided by a larger

	number. However, pursue this so that children see why this cannot give the same answer as the original way round - because they don't have enough objects to share out. Lastly, spend time exploring fact families for mult and division e.g. give a fact and see what other facts we automatically know
Activities	**Show me what you know**
ITP: Grouping http://www.taw.org.uk/lic/itp/grouping.html ITP: Twenty Cards http://www.taw.org.uk/lic/itp/twenty_cards.html ITP: Number Grid http://www.taw.org.uk/lic/itp/num_grid.html BBC Number Jumbler Game - (writing sentences) http://www.bbc.co.uk/schools/starship/maths/games/number_jumbler/small_sound/standard.shtml BBC: How Many Chairs (video) http://www.bbc.co.uk/learningzone/clips/problem-solving-how-many-chairs-are-needed/1799.html	Click here to access files in Google drive