Stage 3	Unit 1: Pattern Sniffing	
Stage 2 support overview	**Stage 3 core learning overview**	**Stage 4 extension overview**
➢ count in steps of 2, 3, and 5 from 0, and in tens from any number, forward and backward ➢ order and arrange combinations of mathematical objects in patterns and sequences ➢ recognise odd and even numbers ➢ recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables	➢ count from 0 in multiples of 4, 8, 50 and 100; ➢ find 10 or 100 more or less than a given number ➢ recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables	➢ **count in multiples of 6, 7, 9, 25 and 1000** ➢ **find 1000 more or less than a given number** ➢ **recall multiplication and division facts for multiplication tables up to 12 × 12** ➢ **recognise and use factor pairs and commutativity in mental calculations**

Key learning steps	Key Vocabulary
1. I can count in steps of 4 and 8 from 0; I can explain how the pattern of 4s and 8s are related 2. I can count in 50s and 100s from 0; I can explain how the pattern of 50s and 100s are related. 3. I can find 10 more and 10 less than a given number 4. I can find 100 more and 100 less than a given number 5. I can count forwards and backwards in tenths, saying the whole number for every ten tenths 6. I can recall the 3 times table (multiplication and division facts) 7. I can recall the 4 and 8 times tables (multiplication and division facts) 8. I can use the 3, 4 and 8 times table facts to solve problems	count forwards/backwards pattern tenth equivalent whole times table multiplication division multiple

Show me... , And another ...	Convince me	What's the same? What's different? (Odd one out)	Always, sometimes, never
10 more than 57/97/403/999 10 less than 43/103/1001 100 more than 432/709/999 100 less than 432/709/2007 ...the fact family for 4x3 ... The 8 times table	...that 267 is ten more than 257 ...the sequence eight tenths, nine tenths, ten tenths could be said as eight tenths, nine tenths, one ... 4x8 gives me the same answer as 8x4	4, 3, 12, 8 3x2, 2x3, 6÷2, 6÷3 16, 24, 32, 44	When I find 10 more than a number, only one digit will change Multiples of three have digits that add up to 3, 6 or 9 Numbers that end in 4 or 8 are multiples of 4.

Misconceptions	Guidance

Pupils forget to include 0 Pupils don't realise that ten tenths is equivalent to a whole number Pupils think that multiples of 3 will all be odd Pupils think that numbers ending in 3 will be multiples of 3 and so on	10 more and 10 less: begin using a 100-square to understand how 10 more is 1 row down and 10 less is 1 row up but then move towards being able to say 10 more or 10 Use the fact family concept to get children to find the associated facts e.g. for 3x4=12 you would also write 4x3=12, 12÷4=3, 12÷3=4. Use a counting stick to help children learn their times tables (multiplication and division facts) as well as to start to see how they relate to each other.
Activities	**Show me what you know**
Compare numbers to 100 NRICH: Sort Them Out (1) * NRICH: Domino Sequences * NRICH: Domino Number Patterns ** NRICH: Next Domino * NRICH: 100 Square Jigsaw * NRICH: That Number Square! *	Click here to access files in Google drive

Stage 3	Unit 2: Investigating Number Systems	
Stage 2 support overview	**Stage 3 core learning overview**	**Stage 4 extension overview**
➢ read and write numbers to at least 100 in numerals and in words ➢ recognise the place value of each digit in a two-digit number (tens, ones) ➢ identify, represent and estimate numbers using different representations, including the number line ➢ use place value and number facts to solve problems ➢ compare and order numbers from 0 up to 100; use <, > and = signs	➢ read and write numbers up to 1000 in numerals and in words ➢ recognise the place value of each digit in a three-digit number (hundreds, tens, ones) ➢ identify, represent and estimate numbers using different representations ➢ solve number problems and practical problems involving these ideas ➢ compare and order numbers up to 1000	➢ **read Roman numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of zero and place value** ➢ **recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones)** ➢ **identify, represent and estimate numbers using different representations** ➢ **solve number and practical problems that involve all of the above and with increasingly large positive numbers** ➢ **round any number to the nearest 10, 100 or 1000** ➢ **round decimals with one decimal place to the nearest whole number** ➢ **order and compare numbers beyond 1000** ➢ **compare numbers with the same number of decimal places up to two decimal places**

Key learning steps	Key Vocabulary	
1. I can read and write numerals and words from 100 to 500 2. I can read and write numerals and words up to 1000. 3. I can read and write decimals with one decimal place using numerals and words. 4. I can understand place value of each digit in a 3 digit number. 5. I can partition 3-digit numbers into hundreds, tens and ones and then in different ways 6. I can position / estimate numbers up to 1000 on a number line or other representation 7. I can solve number problems using representations and known number facts. 8. I can compare two numbers up 1000 and use the signs <, > (and =) to show this comparison.	thousand (s) four digit exact position order most least more than greater than	estimate position hundreds, tens, ones

			less than estimate position hundreds, tens, ones
Show me... , And another ...	**Convince me**	**What's the same? What's different? (Odd one out)**	**Always, sometimes, never**
...the number six hundred and thirty-two in symbols ...the number 405 in words ...how we can represent the number 351 using - base 10 - dienes rods - place value counters - other objects e.g. coins - the number line ...where 350 would be on this blank paper strip that goes from 0-1000? And now where it would be if the strip went from 0-500? 0 -400? ... the number that is ten more than 397? ten less than 548? ... which is the greatest? the least? 243, 342, 432, 234, 423, 324? ... a number that could complete 567 > .... a number that could complete 456 < ...... < 516	... that 324 is less than 342 ... that 567 is represented by 5 hundreds, 6 tens and 7 ones in this apparatus ... that 300 + 100 + 50 + 10 + 2 is a correct partitioning of 462	761, 167, 176, 671, 716, 617 1, 10, 100, 1000 three hundred and five, five hundred and thirty, three hundred and fifty, fifty-three 654 < 765 ; 714 > 704 ; 914 < 940; 435 > 453	There is one way to partition a three digit number There is a 'best way' to partition a three digit number. If you take a three digit number and reverse its digits, you will get a bigger number than you started with. If you take three digits, there are six different three-digit numbers that you can make with them.

Misconceptions	Guidance
Children sometimes write eight hundred as 8 100 Children struggle if either the tens or the units are 'missing' e.g. seven hundred and four can be miswritten as 74 or 740 Children struggle when bridging the hundreds e.g. finding ten less than 407 can lead to wrong answers such as 307 Children confuse the meaning of < and >, finding it hard to tell which is which.	When teaching place value use practical resources to expand on different base representations to emphasise the unitised structure of number ie 231 = 2 hundred squares, 3 ten rods and 1 unit/ ones in Base 10. Vary the resource used here - consider Dienes rods, (place value) counters or even coins. It is important that children develop their number sense here- they should be able to place numbers on a blank number line including where the scale changes. Try taking a blank paper strip as a scale from 0-1000 and asking children to place 200 on it. Then change the scale to 1-500 and ask them to do the same thing - they should be developing the ability to change the placement based on the scale.
Activities	**Show me what you know**
NRICH: Take Three Numbers * NRICH: Three Neighbours ** NRICH: Prison Cells ** NRICH: Spot Thirteen * NRICH: Square Subtraction *** NRICH: Planning a School Trip * NRICH: Magic Vs ** NRICH: Number Differences * NRICH: Sitting Round the Party Tables * NRICH: Dotty Six *	Click here to access files in Google drive

Stage 3	Unit 3: Solving Calculation Problems	
Stage 2 support overview	**Stage 3 core learning overview**	**Stage 4 extension overview**
➢ recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 ➢ "add and subtract numbers using concrete objects, pictorial representations, and mentally, including: - a two-digit number and ones - a two-digit number and tens - two two-digit numbers - adding three one-digit numbers" ➢ show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot	➢ "add and subtract numbers mentally, including: ➢ - a three-digit number and ones ➢ - a three-digit number and tens ➢ - a three-digit number and hundreds" ➢ add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction ➢ estimate the answer to a calculation and use inverse operations to check answers	➢ **add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate** ➢ **multiply two-digit and three-digit numbers by a one-digit number using formal written layout** ➢ **estimate and use inverse operations to check answers to a calculation**

Key learning steps	Key Vocabulary	
1. I can add and subtract ones to and from a three digit number mentally 2. I can add and subtract tens to and from a three digit number mentally 3. I can add and subtract hundreds to and from a three digit number mentally 4. I can add two three digit numbers using a columnar method 5. I can subtract two three digit numbers using a columnar method 6. I can estimate the answer to a three digit + three digit calculation 7. I can estimate the answer to a three digit + three digit or a three digit - three digit calculation 8. I can use the inverse operation to check the answers to addition and subtraction calculations	ones tens hundreds mental mentally add sum of total subtract take away minus less than	more than calculate digit columnar column addition column subtraction estimate inverse operation check

Show me... , And another ...	Convince me	What's the same? What's different? (Odd one out)	Always, sometimes, never
... two numbers that are easy to add	... that if I add a multiple of 100 to this		"A three digit number add a three

... two numbers that are hard to add ... two numbers that are easy to subtract ... two numbers that are hard to subtract ... two numbers with a sum of 220 ... two numbers with a sum of 170 ... two numbers with a sum of 500 ... two numbers with a difference of 200	number, the tens and ones digits will stay the same. ... that 4 hundred and thirty-fourteen is worth the same as 444	130, 250, 360, 135	digit number equals a six digit number A three digit number subtract a three digit number equals a double digit number Addition makes a number larger Subtraction makes a number smaller"

Misconceptions	Guidance
When adding/subtracting 1s, 10s or 100s mentally, children may 'change' the digit in the wrong column. When performing columnar addition, children may forget to include the tens or hundreds they have generated from earlier exchanges. They may also fail to exchange them at all and thus end with a two-digit numbers in the 1s column. When performing columnar subtraction, children may exchange from the wrong column or fail to exchange altogether (instead just finding the difference between the digits in the column, even where the second one is greater than the first). Children may also fail to correctly record the exchange and thus not reduce the tens, for example, by one so that the answer is 10 too high. Some children may use the incorrect operation when checking and fail to realise that they need to use the inverse - this is more pronounced when subtracting.	Encourage children to look at the number in a calculation to decide if they can do it in their head, with jottings or whether they need to use a written method. As in Stage 2 (see guidance here), it is crucial that children see the column addition and subtraction methods as short-hand for the practical process of addition/subtraction with objects. At this level you should aim to place value counters with children but if you need to, go back to objects where the value of the numbers is more obvious e.g. dienes or numicon. Ensure children are going through the full exchange process when adding or subtracting i.e. picking up 10 one counters and swapping them for a ten counter or vice versa. They should then 'regroup' and ensure that the tens and ones are in the right columns to be combined. Encourage children to notate the calculation at the same time as they do it practically - see the videos at the NCETM for examples of this. https://www.ncetm.org.uk/resources/40532 Language is critical in this learning process - make sure you use and insist on the correct terminology for place value e.g. 123+456 would involve twenty add fifty, not two add five. Also insist on children describing their steps orally e.g. I need to add seven ones and 5 ones which makes twelve ones. So I will exchange 10 of these ones for a ten and regroup (put the ten in the right

	column). To help with setting out calculations in columns use large squared paper or laminated grids and mini-WB pens.
Activities	**Show me what you know**
NRICH: How Do You See it? * NRICH: Swimming Pool* NRICH: First Connect Three * NRICH: Sea Level * NRICH: A Bit of a Dicey Problem ***	Click here to access files in Google drive

Stage 3	Unit 4: Exploring Shape	
Stage 2 support overview	**Stage 3 core learning overview**	**Stage 4 extension overview**
➢ "identify and describe the properties of ➢ 2-D shapes, including the number of sides and line symmetry in a vertical line" ➢ "identify and describe the properties of ➢ 3-D shapes, including the number of edges, vertices and faces" compare and sort common 2-D and 3-D shapes and everyday objects	➢ identify horizontal and vertical lines and pairs of perpendicular and parallel lines ➢ recognise angles as a property of shape or a description of a turn ➢ identify right angles, recognise that two right angles make a half-turn, three make three quarters of a turn and four a complete turn; identify whether angles are greater than or less than a right angle	➢ **identify lines of symmetry in 2-D shapes presented in different orientations** ➢ **identify acute and obtuse angles and compare and order angles up to two right angles by size** ➢ **compare and classify geometric shapes, including quadrilaterals and triangles, based on their properties and sizes**

Key learning steps	Key Vocabulary	
1. I can identify horizontal and vertical lines. 2. I can identify pairs of perpendicular lines. 3. I can identify pairs of parallel lines. 4. I can identify the number of angles within 2d shapes. 5. I can use angles to describe a turn. 6. I can identify right angles. 7. I can recognise right angles within a half, three quarter and complete turn 8. I can identify whether angles are greater than or less than a right angle.	horizontal vertical parallel pair perpendicular angle turn direction right angle half quarter	three quarter clockwise anticlockwise complete whole greater than less than position

Show me... , And another ...	Convince me	What's the same? What's different? (Odd one out)	Always, sometimes, never
... a vertical line ... a horizontal line in this picture ... some parallel lines	... that this is a quarter turn to the left ... that this shape has no right angles ... that this shape has more than 6 right angles	parallel and perpendicular horizontal and vertical circle, triangle, square, sphere	2 lines can be parallel and perpendicular at the same time There are no shapes with exactly one right angle

... two lines or faces that are perpendicular to each other ... a shape which has both parallel lines and perpendicular lines ... a shape with a right angle? exactly one right angle? no right angle? ... a quarter turn clockwise/to the right	... that this angle is more than a right angle ... that a triangle cannot have two parallel sides.	right, left, clockwise, anticlockwise	

Misconceptions	**Guidance**
Children sometimes confuse horizontal and vertical directions and lines. Children sometimes have a lack of understanding of pairs as implying a set of 2. The concept of turn is confusing for some children as it requires measurement in a context not often used - comparison to a whole turn is the easiest way into this e.g. quarter turn before right angle. Weak understanding of halves, quarters and three quarters can limit reasoning about turn. Children may interpret a 'right' angle as a right hand turn. They also believe that right angles are always flat and struggle to see them in 3D space. When a right angle is oriented off the horizontal/vertical, children may not recognise it as such. They often infer that the definition of a right angle involves one line being parallel with the bottom of the page. Children sometimes think that parallel lines move away from each other/towards each other and can be confused by images showing perspective e.g. railway tracks into the distance. The definition of parallel lines as 'not meeting' may not be as helpful as the	Explore the meaning of horizontal and vertical and provide real life examples. Engage children in practical/kinesthetic activities to help them remember these words. Consider the order of learning perpendicular and right angles as children need to understand right angles in order to fully understand perpendicular. Think carefully about how you define parallel lines - the more popular 'lines that never meet' may be more problematic than the more technical \lines that are always a fixed distance apart'. When teaching about angle and turn, bear in mind that this may be unfamiliar to many children. Try to avoid using 'corner' interchangeably with angle or turn as they are different concepts - a corner is a point in space whereas an angle is a turn through space. Compare turns initially to a complete or whole turn, moving on to thinking about half turns and quarter turns. Use directions such as 'to the right' or 'to the left' initially but be prepared to use words like clockwise and anticlockwise. Any children who can tell the time may already use this vocabulary themselves and it is more reliable as it is not dependent on where you are relative to others. Once you have defined a right angle as exactly a quarter turn you can link it to the shape of a 'quarter of a circle' to 'see' the perfect 90 degree angle that it

more formal 'lines that are always the same distance apart' for these children.	represents. Encourage children to find these angles everywhere, in 2D and 3D examples. There is a significant amount of terminology in this unit and so time needs to be invested in recalling and using these words to enable them to access the concepts.
Activities	**Show me what you know**
NRICH: Square It * NRICH: Square It *	Click here to access files in Google drive

<table>
<tr><th>Stage 3</th><th colspan="2">Unit 5: Generalising Arithmetic</th></tr>
<tr><th>Stage 2 support overview</th><th>Stage 3 core learning overview</th><th>Stage 4 extension overview</th></tr>
<tr>
<td>➢ recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100
➢ "add and subtract numbers using concrete objects, pictorial representations, and mentally, including:
- a two-digit number and ones
- a two-digit number and tens
- two two-digit numbers
- adding three one-digit numbers"
➢ recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.
➢ show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot</td>
<td>➢ "add and subtract numbers mentally, including:
➢ - a three-digit number and ones
➢ - a three-digit number and tens
➢ - a three-digit number and hundreds"
➢ add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction
➢ solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction</td>
<td>➢ add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate
➢ multiply two-digit and three-digit numbers by a one-digit number using formal written layout
➢ use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
➢ solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why</td>
</tr>
<tr><th colspan="2">Key learning steps</th><th>Key Vocabulary</th></tr>
<tr>
<td colspan="2">1. I can add and subtract ones to and from a three digit number mentally
2. I can add and subtract tens to and from a three digit number mentally
3. I can add and subtract hundreds to and from a three digit number mentally
4. I can add two three digit numbers using a column method
5. I can subtract two three digit numbers using a column method
6. I can use number facts and place value to solve missing number problems
7. I can solve more complex addition problems
8. I can solve more complex subtraction problems</td>
<td>ones
tens
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calculate
digit
column addition
column subtraction
facts
place
value
inverse
operation
check</td>
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Show me... , And another ...	Convince me	What's the same? What's different? (Odd one out)	Always, sometimes, never
...two numbers with a sum of 230 - and another two ... ... 435 + 100? + 200? + 500? ... 256 + 99? + 9? ... how you can add 567 + 678 - using dienes rods? - using place value counters? - using column method? ... two numbers with a sum of 20 and a difference of 12	... that 542 + 100 - 1 gives the same answer as 542 + 99 ... that 123 + 456 gives the same answer as 156 + 423 ... that order matters when you are subtracting ... 91 – 74 does not equal 23 (90 - 70 - 20, 4 - 1 =3) ...2 three digit numbers with a sum of 473	234 + 100; 235 + 99; 236 + 98; 244 + 90 564 - 213; 563 - 212; 562 - 211; 554 - 203	Two numbers will have the same sum if you increase one by 7 and decrease the other by 7 Two numbers will have the same difference if you increase one by 7 and decrease the other by 7

Misconceptions	Guidance
Children struggle to add and subtract from right to left in columns - if the numbers are partitioned this does not affect addition but it will impact subtraction and cause an issue with exchanging tens etc. Children may place the smallest number at the top of the calculation when using column subtraction When numbers are carried or borrowed, children may forget to include them in the new calculations. Look out for children apparently carrying out column method fine for addition and subtraction but struggling whenever they have to carry/exchange - this indicates a lack place value understanding. When attempting missing number problems e.g. 45 + ? = 100 children may	When teaching missing number problems, model this using a bar model or a number line so they can see what happens and make a decision about the operation they need to carry out before doing so. Encourage children to count efficiently, ie in ones to the nearest ten then on in tens to the answer or to the nearest hundred. Ensure children understand carrying and borrowing when using column addition and subtraction - use place value models to support this process before going to a written method. The calculation policy expects that children will carry out addition and subtraction using dienes rods and then place value counters before formally recording as columns. It is useful to do the 'written' method alongside the practical method so that children can directly see how their actions are recorded within the column

give the answer 65 as they have counted up in tens then ones	format. Distinguish between calculations that require carrying or borrowing and those that don't. Use correct terminology for place value e.g. 123+456 would involve twenty add fifty, not two add five. Use visual representation to help show children what happens to numbers when adding or subtracting ones, tens and hundreds
Activities	**Show me what you know**
NRICH: A Square of Numbers * NRICH: Buying a Balloon * NRICH: GOT IT ** NRICH: Make 37 ** NRICH: Consecutive Numbers ** NRICH: Super Shapes * NRICH: Strike it Out *	Click here to access files in Google drive

Stage 3	Unit 6: Reasoning with Measures	
Stage 2 support overview	**Stage 3 core learning overview**	**Stage 4 extension overview**
➢ recognise and use symbols for pounds (£) and pence (p); combine amounts to make a particular value ➢ find different combinations of coins that equal the same amounts of money ➢ solve simple problems in a practical context involving addition and subtraction of money of the same unit, including giving change	➢ add and subtract amounts of money to give change, using both £ and p in practical contexts ➢ measure the perimeter of simple 2-D shapes	➢ **estimate, compare and calculate different measures, including money in pounds and pence** ➢ **measure and calculate the perimeter of a rectilinear figure (including squares) in centimetres and metres** ➢ **find the area of rectilinear shapes by counting squares**

Key learning steps	Key Vocabulary
1. "I can add and subtract amounts of money using £ and p. 2. I can compare different amounts of money, saying which is greater than or less than." 3. I work out how much change to give by subtracting the total cost from the amount given 4. I can use £ and p in practical contexts such as a pretend shop, going to a real shop or buying things online 5. I can find the perimeter of a shape drawn on cm squared paper by counting or by counting and adding 6. I can compare the sizes of objects by measuring them in standard units. I can compare the weights of objects with standard weights using a balance. 7. I can measure and draw straight lines in whole numbers of cm using a ruler and apply it to practical problems, such as cutting out a rectangle of a given size 8. I can measure the perimeter of simple 2D shapes by measuring the sides and adding their lengths 9. I can solve simple perimeter problems, such as total length of fence needed for a field.	balance; buy; centimetre, cm; draw; greater; length; less; perimeter; problem; rectangle; ruler; shape; side; size; solve; square; squared; paper; straight; line; unit; weight

Show me... , And another ...	Convince me	What's the same? What's different? (Odd one out)	Always, sometimes, never
Make me a 7cm by 5cm rectangle from a piece of card Accurately measure the size of an A4 sheet.	That a 6cm by 6cm square has the same perimeter as a 7cm by 5cm rectangle.	Compare a 10cm by 7cm rectangle with an L shape made by cutting a 3cm square from one corner. (Same perimeter, still all right angles, different area, different shape)	If I make a triangle by halving a rectangle, the triangle perimeter is half of the rectangle perimeter

Misconceptions	Guidance
Ignoring units so 5p > £2; lack of arithmetic skills; counting border squares (ie including corners) or dots (on dotty paper) rather than counting cm; failure to comprehend perimeter; not measuring from 0; not ensuring scales balance;	Children should be given the chance to run a play shop, 'sell' things, add up the total and give change. The work on measurement begins with counting the cm along the edge of a shape. The ruler speeds things up by allowing an

lack of fine motor skills; inability to keep pencil hard against ruler	answer without counting, but the two methods need to run side by side long enough to establish the link. In the event of difficulty, go back to counting with the ruler alongside. Note the importance of developing measuring skills and drawing to a given size. Perimeter is established as the total distance round. This can be obtained by measuring each side and adding but also by 'adding on the ruler' (eg for a 4 x 3 rectangle, the first side is measured as 4, the second side is measured from 4 to 7, then 7 to 11 then 11 to 14). Both should be experienced to develop understanding. Children need to gain experience in using weights/lengths if the measures are to be meaningful and also as the basis of future estimation.
Activities	**Show me what you know**
	Click here to access files in Google drive

Stage 3	Unit 7: Discovering Equivalence	
Stage 2 support overview	**Stage 3 core learning overview**	**Stage 4 extension overview**
➢ recognise, find, name and write fractions 1/3, 1/4, 2/4 and 3/4 of a length, shape, set of objects or quantity ➢ write simple fractions for example, 1/2 of 6 = 3 and recognise the equivalence of 2/4 and ½ ➢ identify, represent and estimate numbers using different representations, including the number line	➢ recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators ➢ recognise and use fractions as numbers: unit fractions and non-unit fractions with small denominators ➢ compare and order unit fractions, and fractions with the same denominators ➢ recognise and show, using diagrams, equivalent fractions with small denominators ➢ identify, represent and estimate numbers using different representations	➢ **recognise and show, using diagrams, families of common equivalent fractions** ➢ **count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10** ➢ **recognise and write decimal equivalents of any number of tenths or hundredths** ➢ **recognise and write decimal equivalents to 1/4,1/2, 3/4**

Key learning steps
1. I can find unit fractions of a discrete set of objects using a division process. 2. I can find simple non-unit fractions of a discrete set of objects. 3. I can write an amount of objects as a fraction of a whole set 4. I can recognise a unit fraction as a number between 0 and 1 and position it on a number line. 5. I can compare and order unit fractions 6. I can recognise a non-unit fraction as a number between 0 and 1 and position it on a number line. 7. I can compare and order fractions with the same denominator 8. I can recognise two equivalent fractions represented visually; I can create representations to show whether two fractions are equivalent.

Key Vocabulary	
fraction	fifth
unit fraction	sixth
non-unit fraction	eighth
denominator	tenth
numerator	represent
proportion	number line
out of	compare
half	greater than
third	less than
quarter	order
	equivalent

Show me... , And another ...	Convince me	What's the same? What's different? (Odd one out)	Always, sometimes, never
... 1/3 of 21 ... how you can represent 3/4 in as many different ways as possible ... how you could use an array to find 3/5 of 30	... that 1/3 > 1/5 ... finding 1/6 is the same as dividing by 6	1/5, 2/5, 3/5, 4/5 1/2, 1/3, 1/4, 1/5 1/3, 2/6, 1/2, 3/6	... unit fractions cannot be greater than 1/2 ... you cannot compare fractions with different denominators

... if this strip represents 24, where 1/2 would be? 1/4? 1/3? 1/6?	... that there are at least 10 different ways to represent 1/6 ... that you can see the whole fraction family using an array ... that 2/10 is worth the same as 1/5 ... that a unit fraction sits on the number line between 0 and 1	3/4, 1/4 1/4 1/4, three quarters, 6/8	... you can order fractions with the same denominator by just putting the numerators in order ... there is no equivalent fraction to 4/5 ... 3/3 is the same as one whole ... a fraction is a number AND a proportion of a shape ... fractions will always sit below 1 on the number line

Misconceptions	**Guidance**
Children may not have a complete understanding of a fraction as both a proportion of the whole (be that one whole object or a quantity or a set of objects) as well as a number in its own right. Children often think that fractions with a larger denominator are greater than fractions with a smaller one e.g. they may say that 1/10 > 1/5 because 10 > 5. When considering unit fractions, pupils may not understand that each 'part' needs to be the same size Pupils often interchange the denominator and numerator Look out for international language e.g. the use of a 'fourth' for a quarter. Similarly, listen to how children pronounce non-unit fractions e.g. 2/3 is sometimes said as '2 over 3' or '2 threes' rather than a more valid alternative such as 'two thirds'.	It is absolutely essential that by this stage of learning children understand the role of the denominator (and then the numerator). i.e. the denominator shows how many equal parts a number, or an object, or a set of objects, has been divided into, while the numerator (top number) tells us how many of those parts there are. Look out for children who do not divide a quantity or object into EQUAL parts as these children do not fully understand the meaning of the fraction. Spend some time on representing fractions in multiple ways - always model more than one representations and ask children to develop their own. These can include: area diagrams using a range of different shapes, number lines, words, symbols, some decimal equivalents and percentages, fractions as a result of division. Pictorial representations of a particular fraction may be of different sizes and different shapes. For example, don't always use shaded sections of circles, and interesting discussions can be had from drawing half of a small square and a quarter of a larger square and asking which is the larger fraction (and this means you have to be careful when using areas to explain fractions!).

When comparing fractions, pupils may be swayed by a numerator to determine size e.g. they may say that 2/6 > 1/3 because 2 > 1 with no consideration for the different denominators Children tend to become attached to one or two representations of a fraction and this limits their ability to transfer their knowledge to fractions as 'proportions of' or fractions as 'numbers' or fractions as 'a process of sharing' e.g. 2/3 as 2 shared by 3. They may select an inappropriate representation to assist them with a given problem.	There is a key learning point that the rest of KS2 and KS3 depends on that emerges in Stage 3 - that is the difference between fractions as ordinal numbers (as numbers on a number line), fractions as being a special kind of cardinal number (the answer to 1/2 of a number depends on the quantity you are using) and fractions as operators (What is ½ of 30? What is 2/3 of 45?). In Stage 3 we are especially developing children's understanding of a fraction as an actual number between 0 and 1 (for proper fractions). Make connections with a range of representations here to show how dividing the number line between 0 and 1 enables you to 'see; where 1/3 must sit for example. Try using a bead string as a precursor to a number line to give some sense of cardinality to the process (so the children are finding 1/5 of 20 for example) before asking them to think of 1/5 of an abstract length). This is a good time to introduce paper strips for children to start trying to place fraction appropriately on the strip. You can use a paperclip or a finger to show their responses. Start by giving the strip a value e.g. the strip represents a length of 12 and ask children to show where 1/2 would be, 1/4 and so on. They will initially do this by finding 1/2 of 12 and so but will eventually begin to see that the fraction itself determines the position. You can then move on to a number line more formally. This links well to the use of the Bar Model to represent fractions of amounts particularly - see link https://www.ncetm.org.uk/resources/42194 - and if you are using this representation to model calculations with addition, subtraction etc then it is recommended that you maintain consistency and also apply it to these non-integer contexts. The array representation is a useful way of showing both unit fractions but then their family of fractions too - get children to think about the 'fifth' family of 30, for example, by arranging 30 counters into an array with 5 rows (they should count out 5 counters each time before starting a new column). Then explore how the array 'shows' the value of 1/5 of 30 as well as 2/5 and so on. When comparing and then ordering (which is just comparing with more than two items!), use the number line or array representations to give you best access - this way children can see which fraction is larger once they have been

	positioned in a comparable way. You might want to start by showing two LESS useful comparative representations to get children to realise the difficulties for themselves.
Activities	**Show me what you know**
"Maths4Life Sharing Cakes https://www.ncetm.org.uk/resources/m4l_fractions_1 NRICH: Fractional Triangles http://nrich.maths.org/2124/note NZ Maths TRAINS (for fractions as operators AND numbers) http://www.nzmaths.co.nz/resource/trains NRICH: Fractional Walls (for discovering equivalent fractions) http://nrich.maths.org/4519 Fraction models and support questions - http://www.annery-kiln.eu/gaps-misconceptions/all-images.html	Click here to access files in Google drive

<table>
<tr><th colspan="2">Stage 3</th><th colspan="2">Unit 8: Investigating Statistics</th></tr>
<tr><th>Stage 2 support overview</th><th>Stage 3 core learning overview</th><th colspan="2">Stage 4 extension overview</th></tr>
<tr><td>➢ interpret and construct simple pictograms, tally charts, block diagrams and simple tables
➢ ask and answer simple questions by counting the number of objects in each category and sorting the categories by quantity
ask and answer questions about totaling and comparing categorical data</td><td>➢ interpret and present data using bar charts, pictograms and tables
➢ solve one-step and two-step questions [for example, ‘How many more?’ and ‘How many fewer?’] using information presented in scaled bar charts and pictograms and tables</td><td colspan="2">➢ interpret and present discrete and continuous data using appropriate graphical methods, including bar charts and time graphs
➢ solve comparison, sum and difference problems using information presented in bar charts, pictograms, tables and other graphs</td></tr>
<tr><th colspan="2">Key learning steps</th><th colspan="2">Key Vocabulary</th></tr>
<tr><td colspan="2">1. I can record data from an experiment or survey directly in a tally chart
2. I can produce a frequency table from a list of data or a tally chart
3. I can construct a pictogram for my data where each symbol represents more than one object
4. I can construct a bar chart for my data using a scale that goes up in 1s
5. I can construct a bar chart for my data using a scale that goes up in 2s or 10s
6. I can read information from a pictogram where each symbol represents more than one object.
7. I can read information from a bar chart where the scale goes up in 2s, 5s or 10s.
8. I can answer complex questions about my data using my readings from a bar chart or pictogram.</td><td>data
category(ies)
pictogram
key
symbol
represents
tally
bundle
frequency
frequency table
total frequency
scale</td><td>interval
bar chart
how many more ...
(less)
difference
total
popular
common
rare
more than
fewer than</td></tr>
</table>

Show me... , And another ...	Convince me	What's the same? What's different? (Odd one out)	Always, sometimes, never
... how you would represent three objects using this symbol (symbol worth 2, 4 etc.) ... how many these symbols represent (pictogram symbols worth more than 1 inc partial symbols) ... the frequency table for this data	... that it is not a good idea to let my symbol represent 7 objects in my pictogram ... that it is quicker to draw a pictogram when each symbol represents more than one object	bar chart and block graph tally and list 2s, 5s, 7s, 10s as amounts to go up in for scale	... the highest value on a bar chart is the best ... a pictogram is easier to read than a tally chart ... a tally chart is easier to read than

<table>
<tr><td>set
... the frequency table that this bar chart came from
... the tally chart that this pictogram may have come from</td><td>... that the most popular answer was .....

... the best way to display the data is using a bar chart</td><td></td><td>a list

... you can find the exact list of data that a bar chart came from</td></tr>
<tr><th colspan="2">Misconceptions</th><th colspan="2">Guidance</th></tr>
<tr><td colspan="2">Children often record their data in a haphazard way and hence make counting errors when converting to tally charts or tables. This is made worse when they do not use a piece-by-piece approach to tallying the data and cross off the data as it is used.

When working with pictograms with symbols worth more than 1, children sometimes struggle to draw and/or to interpret partial symbols to represent numbers that are not exact multiples of the symbol value. They may also assume that a symbol is worth one without checking a key. Similarly, they may forget to include a key when constructing.

Children similarly struggle with bar charts where the scale does not go up in 1s to estimate the value of a given bar. This is exacerbated when the scale is in 10s or a larger unit.

When interpreting, children often fail to relate the number to the context, assuming that a higher number is always the best and not using language related to the situation.</td><td colspan="2">The emphasis here is on working with data representations where there is not a 1:1 correspondence between the symbols/scales used. This is a good opportunity to link to times table practice as well as to explore why scales that go up in 2s, 4s, 5s, 10s etc are greatly preferred to those using 3s, 6s and so on.

Make sure children have the opportunity to collect their own data before presenting it in the different formats. Discuss the advantages and disadvantages of each approach. Also allow time for analysis of data that has already been presented - what is critical here is that children can do more than read the figures from the charts and can instead INTERPRET the information in the context of the problem. E.g. if the chart shows 100m race times, which outcome is the most desirable?

It is also well worth asking children to work backwards and construct the frequency table (and maybe even the original data list) from the chart itself.</td></tr>
<tr><th colspan="2">Activities</th><th colspan="2">Show me what you know</th></tr>
<tr><td colspan="2"></td><td colspan="2">Click here to access files in Google drive</td></tr>
</table>

Stage 3	Unit 9:Solving Number Problems	
Stage 2 support overview	**Stage 3 core learning overview**	**Stage 4 extension overview**
➢ calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs ➢ show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot ➢ solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts	➢ write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods ➢ solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects	➢ **find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths** ➢ **"use place value, known and derived facts to multiply and divide mentally, including:** ➢ **multiplying by 0 and 1;** ➢ **dividing by 1;** ➢ **multiplying together three numbers"** ➢ **multiply two-digit and three-digit numbers by a one-digit number using formal written layout** ➢ **solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects** ➢ **solve simple measure and money problems involving fractions and decimals to two decimal places**
Key learning steps		**Key Vocabulary**

1. I can use times table facts to write multiplication and division calculations.. 2. I can find missing numbers in simple multiplication and division problems. 3. I can multiply a multiple of 10 by a single digit. e.g. 20 x 3. 4. I can multiply a 2d by a 1d (based on known times tables) using mental methods 5. I can multiply a 2d by a 1d using partitioning and the grid method. 6. I can solve multiplication problems involving 2d numbers. 7. I can solve scaling problems involving either multiplication or division. 8. I can solve correspondence problems where n objects are connected to m objects.	multiplication, division times table facts fact family partition array grid solve problem scaling correspondence time larger/smaller

Show me... , And another ...	Convince me	What's the same? What's different? (Odd one out)	Always, sometimes, never
... the fact family for 6 x 5 ... the missing number: 3 x ☐ = 18, ☐ x 2 = 24 20 ÷ ☐ = 5, ☐ ÷ 3 = 7 ... how you could represent 13 x 5 with an array ... the fact family for 17 x 3 ... how you can represent this problem visually: Jane is drawing a picture of a flower. Her picture is 4cm tall. The real flower is 3 times as tall as this. How tall is the real flower? ... how you can represent this problem visually: Alice has 6 boxes. In each box there are 4 sweets. How many sweets does Alice have	... 15 x 10 = 150 ... 23 x 4 is the same as 20 x 4 + 3 x 4 ... when you know a times table fact, you actually know 4 facts	7 x 3, 3 x 7 , 3 x 21, 21 ÷ 3, 21 ÷ 7, 21 x 7, 7 ÷ 3, ... 14 x 2, double 14, the number twice as big as 14, 28 multiply, lots of , groups of, divide, product, shared by, shared between, grouped into, quotient	... multiplying is the opposite of dividing ... every times table fact has two related division facts ... the opposite of multiplying by 3 is finding a third ... if you know that a x b = c then you also know that a ÷ b = c

altogether? ... how you can represent this problem visually: Jay has 18 hats. He shares the hats equally into 3 bags. How many hats will there be in each bag?			

Misconceptions	Guidance
Children may struggle to correctly rearrange the numbers of a times table fact into the corresponding division fact, for example they may put the smaller numbers first in division calculations. Some children may not yet have a strong understanding that multiplication is the inverse of division and so find it hard to move between the two operations. Children with weak understanding of how to represent a multiplication as an array may struggle to represent and calculate a 2dx1d multiplication and fail to see why they need to be able to partition it. They may try to work with a very large array rather than sectioning it off and partitioning. Poor understanding of place value may affect a child's ability to partition correctly and to calculate the result of a multiple of 10 x 1d. Some children struggle to understand the range of language of multiplication and division e.g. they may forget the meanings of 'lots of', 'times bigger', 'product' etc. Children often fail to recognise scaling problems as multiplication (or division problems) and find it hard to represent these practically.	Times table: Make sure that children don't just count forwards in multiples and that they have lots of exposure to counting back as well as forward. When teaching or rehearsing times tables make the link to division very clear. Encourage children to procude the fact family for a given times table fact (i.e. the commutative multiplication fact plus the two related division facts). When exploring multiplying a multiple of 10 by a 1d, try to use the unitisation skills of place value. For example, 40 x 5 is 4 tens multiplied by 5 which is 20 tens or 200. You can then gradually move towards the 'short cut' rather than directly teaching it without any concept behind. Ensure arrays are really secure as representations before exploring 2d x 1d. Children need to be able to independently produce the array for 14 x 3 before you show them a nifty way of partitioning it into a 10x3 and a 4x3 array stuck together. They need to see partitioning as a physical process applied to the array BEFORE you move to partitioning as an abstract process applied to just a number. You can then quite quickly progress to the grid method as an array without every piece showing and, for the most able, you can even look at a column representation for this. Throughout the unit, make sure that the language of calculation is both modelled and displayed - ensure you use ALL OF multiplication, times, lots of, groups of, multiply, product, times bigger, times as big etc. There is a strong focus on applying the skills above to problem solving in this

	unit - children need to see problems that are based on BOTH repeated addition [lots of] AND scaling which is often less developed (hence the final two learning steps). Try using the Bar Model to represent scaling problems to give the children somethign tangible to base their calculations on. Remember, the bar is a way of representing a problem to help you decide which calculations to do so give them lots of practice with this as well as the actual multiplications or dvisions. Don't forget to include division questions within scaling problems also.
Activities	**Show me what you know**
	Click here to access files in Google drive