



Stage 4		Unit 1: Pattern Sniffing	
Stage 3 support overview		Stage 4 core learning overview	
<ul style="list-style-type: none"> ➤ count from 0 in multiples of 4, 8, 50 and 100; ➤ find 10 or 100 more or less than a given number ➤ recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables 		<ul style="list-style-type: none"> ➤ count in multiples of 6, 7, 9, 25 and 1000 ➤ find 1000 more or less than a given number ➤ recall multiplication and division facts for multiplication tables up to 12 x 12 ➤ recognise and use factor pairs and commutativity in mental calculations 	
		<ul style="list-style-type: none"> ➤ count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000 ➤ recognise and use square numbers and cube numbers, and the notation for squared (²) and cubed (³) ➤ multiply and divide numbers mentally drawing upon known facts ➤ identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers ➤ know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers ➤ establish whether a number up to 100 is prime and recall prime numbers up to 19 	
Key learning steps		Key Vocabulary	
<ol style="list-style-type: none"> 1. I can count in steps of 6, 7, 9 from 0 2. I can count in steps of 25 and 100 from 0; I can explain how the pattern of 25s and 100s are related to 100s 3. I can find 1000 more or 1000 less than a given number 4. I can count backwards from a positive number using negative numbers after 0. 5. I can count forwards and backwards in hundredths, saying the whole number for every ten tenths 6. I can give the fact family for any multiplication up to 12x12 (or associated division); I can use these families to solve problems 7. I can find factor pairs of a number using times table facts 8. I can complete mental calculations using factor pairs to help me 		negative hundredth fact family factor factor pair commutative multiple	
Show me... , And another ...	Convince me	What's the same? What's different? (Odd one out)	Always, sometimes, never
...1000 more than 4567/12045	... axb gives me the same answer as	1, 2, 3, 4	When I find 1000 more than a



<p>...1000 less than 4567/12045 ... the fact family for a factor pair that makes 18 ... all the factor pairs that make 20 ... A number with an odd number of factors</p>	<p>bx ... a fact family will always have four facts ... 10 hundredths are equivalent to a tenth</p>	<p>3, 6, 12, 18 167, 1167, 2167, 3167</p>	<p>number, only one digit will change Multiples of 6 are also multiples of 2 and of 3 Numbers in the nine times table have digits that add up to 9</p>
<p>Misconceptions</p>		<p>Guidance</p>	
<p>Pupils don't realise that one hundred hundredths is equivalent to a whole number Or that ten hundredths is equivalent to a tenth</p> <p>Pupils struggle to find 1000 more when bridging a 10,000 - e.g. 1000 more than 9647</p> <p>Pupils forget about 1 and the number itself being factors. Pupils may try to use a non-integer as a factor e.g. 2.5</p>		<p>Try to establish the links between tenths and hundredths when counting in hundredths e.g. ten hundredths could be send as one tenth. Consider using other fraction equivalents also e.g. half and quarter. When counting these ensure you have a visual representation as well as the symbolic representation for the children to 'absorb'</p> <p>Commutativity - ensure that children see models and images to show why multiplication is commutative e.g. an array viewed from two different positions</p> <p>Encourage pupils to find ALL the factor pairs of a number and to find a way of convincing you that they have them all</p>	
<p>Activities</p>		<p>Show me what you know</p>	
<p>Recall multiplication and division facts for multiplication tables up to 12x12 NRICH: Multiplication Square Jigsaw * NRICH: Shape Times Shape * NRICH: Table Patterns Go Wild! ** NRICH: Let's Divide Up! * NRICH: That Number Square! * NRICH: Carrying Cards * NRICH: Light the Lights Again * NRICH: Multiples Grid * NRICH: Zios and Zepts *</p>		<p>Click here to access files in Google drive</p>	



Stage 4		Unit 2: Investigating Number Systems																							
Stage 3 support overview		Stage 4 core learning overview																							
<ul style="list-style-type: none"> ➤ read and write numbers up to 1000 in numerals and in words ➤ recognise the place value of each digit in a three-digit number (hundreds, tens, ones) ➤ identify, represent and estimate numbers using different representations ➤ solve number problems and practical problems involving these ideas ➤ compare and order numbers up to 1000 		<ul style="list-style-type: none"> ➤ read Roman numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of zero and place value ➤ recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones) ➤ identify, represent and estimate numbers using different representations ➤ solve number and practical problems that involve all of the above and with increasingly large positive numbers ➤ round any number to the nearest 10, 100 or 1000 ➤ round decimals with one decimal place to the nearest whole number ➤ order and compare numbers beyond 1000 ➤ compare numbers with the same number of decimal places up to two decimal places 																							
		<ul style="list-style-type: none"> ➤ read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit ➤ read Roman numerals to 1000 (M) and recognise years written in Roman numerals ➤ read, write and interpret negative numbers in context ➤ solve number problems and practical problems that involve all of the above ➤ round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000 ➤ ➤ round decimals with two decimal places to the nearest whole number and to one decimal place ➤ read, write, order and compare numbers with up to three decimal places 																							
Key learning steps		Key Vocabulary																							
<ol style="list-style-type: none"> 1. I can recognise Roman Numerals, identify contexts in which they are used and read/write the numbers 1-10 in Roman Numerals. 2. I can read and write Roman Numerals to 100. 3. I can understand place value of each digit in a 4 digit number as well as partition 4-digit numbers into thousands, hundreds, tens and ones and then in different ways 4. I can round any number to the nearest 10, 100 or 1000 5. I can round decimals with one decimal place to the nearest whole number 6. I can solve number/practical problems with numbers up to 10000. 7. I can order and compare numbers beyond 1000, using the signs <, > (and =) to show this comparison. 8. I can order and compare numbers with up to two decimal places, using the signs <, > (and =) to show this comparison. 		<table style="width: 100%; border-collapse: collapse;"> <tr> <td>Roman Numerals</td> <td>50 L</td> </tr> <tr> <td>numeral</td> <td>100 C</td> </tr> <tr> <td>1 I</td> <td>500 D</td> </tr> <tr> <td>2 II</td> <td>1000 M</td> </tr> <tr> <td>3 III</td> <td>round</td> </tr> <tr> <td>4 IV</td> <td>integer</td> </tr> <tr> <td>5 V</td> <td>decimal</td> </tr> <tr> <td>6 VI</td> <td>decimal places</td> </tr> <tr> <td>7 VII</td> <td>compare</td> </tr> <tr> <td>8 VIII</td> <td></td> </tr> <tr> <td>9 IX</td> <td></td> </tr> </table>		Roman Numerals	50 L	numeral	100 C	1 I	500 D	2 II	1000 M	3 III	round	4 IV	integer	5 V	decimal	6 VI	decimal places	7 VII	compare	8 VIII		9 IX	
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10 X

Show me... , And another ...	Convince me	What's the same? What's different? (Odd one out)	Always, sometimes, never
<p>... that there are exactly ten numbers between 2000 and 3000 with a tens digit of 4 and a ones digit of 9</p> <p>... that $4671 < 4716$</p> <p>... that 69 in Roman Numerals is LXVI</p>	<p>... that 324 is less than 342</p> <p>... that 567 is represented by 5 hundreds, 6 tens and 7 ones in this apparatus</p> <p>... that $300 + 100 + 50 + 10 + 2$ is a correct partitioning of 462</p>	<p>4562, 2654, 6452, 5246, 6254, 2456</p> <p>1, 10, 100, 1000, 10000</p> <p>VI, XVI, LVI, CVI</p> <p>2.7, 3.4, 2.5, 3.9</p>	<p>If you have two decimals, the longer decimal will be worth more than the shorter decimal</p> <p>You cannot show decimals using Roman Numerals</p> <p>If you take 4 digits, there are 24 different 4-digit numbers that you can create from them (development - 4 different digits or no such restriction)</p> <p>There are 9 integers? for which $3567 < ? < 3576$</p>
Misconceptions		Guidance	
<p>Children find it had to adapt to the code of roman numerals and they try to translate place value concepts directly.</p> <p>Children think that 49 is IL - breaking the 'adjacent symbol rule'</p> <p>Children think that 40 is XXXX - breaking the '3 max' rule</p> <p>On clocks, sometimes 4 is written as IIII rather iV for aesthetic reasons - this can be confusing as it breaks the rules!</p> <p>Children read decimals incorrectly saying 'three point forty-two' instead of 'three point four two'.</p> <p>When ordering children think that 'longer' decimals are larger e.g. they presume that $3.14 > 3.4$</p> <p>To develop children's number sense, make use of opportunities to place</p>		<p>When introducing Roman Numerals ensure a whole school approach is adopted, ie on displays around clock faces.</p> <p>The history will need to be explored to unpick 'the rules'.</p> <p>Children need to understand that we are not calculating with Roman Numerals but making connections to real life and how they are represented today. This is just one alternative number system but there are a multitude of others.</p> <p>Good SMSC opportunity.</p> <p>There is a clear link to money with decimals with two d.p. and this can be exploited to help children grasp the concepts of ordering and rounding.</p> <p>Try to use the language of place value with decimals as well as with integers e.g. 4 tenths and 2 hundredths OR 42 hundredths and to use the usual apparatus to represent these numbers in different ways e.g. place value counters</p>	



numbers on a blank scale (e.g. paper clip on blank strip of paper or blank laminated number line)	
Activities	Show me what you know
NRICH: Some Games That May Be Nice or Nasty * NRICH: The Deca Tree *	Click here to access files in Google drive



Stage 4		Unit 3: Solving Calculation Problems																											
Stage 3 support overview	Stage 4 core learning overview	Stage 5 extension overview																											
<ul style="list-style-type: none"> ➤ "add and subtract numbers mentally, including: ➤ - a three-digit number and ones ➤ - a three-digit number and tens ➤ - a three-digit number and hundreds" ➤ add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction ➤ estimate the answer to a calculation and use inverse operations to check answers 	<ul style="list-style-type: none"> ➤ add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate ➤ multiply two-digit and three-digit numbers by a one-digit number using formal written layout ➤ estimate and use inverse operations to check answers to a calculation 	<ul style="list-style-type: none"> ➤ add and subtract numbers mentally with increasingly large numbers ➤ add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction) ➤ multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers ➤ divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context ➤ use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy 																											
Key learning steps		Key Vocabulary																											
<ol style="list-style-type: none"> 1. I can add two numbers up to four digits using a columnar method 2. I can subtract two numbers up to four digits using a columnar method 3. I can multiply a two digit number by a one digit number informally (using practical equipment or a representation to help me). 4. I can multiply a two digit number by a one digit number using a formal written method 5. I can multiply a three digit number by a one digit number using a formal written method 6. I can estimate the answer to addition and subtraction calculations involving four digits 7. I can estimate the answer to multiplication calculations involving four digits 8. I can use the inverse operation to check answers to addition, subtraction or multiplication calculations 		<table style="width: 100%; border: none;"> <tr> <td style="width: 50%;">ones</td> <td style="width: 50%;">minus</td> </tr> <tr> <td>tens</td> <td>less than</td> </tr> <tr> <td>hundreds</td> <td>more than</td> </tr> <tr> <td>thousands</td> <td>multiply</td> </tr> <tr> <td>mental</td> <td>divide</td> </tr> <tr> <td>mentally</td> <td>product</td> </tr> <tr> <td>add</td> <td>calculate</td> </tr> <tr> <td>sum of</td> <td>digit</td> </tr> <tr> <td>total</td> <td>column addition</td> </tr> <tr> <td>subtract</td> <td>column subtraction</td> </tr> <tr> <td>take away</td> <td>estimate</td> </tr> <tr> <td></td> <td>inverse</td> </tr> <tr> <td></td> <td>operation</td> </tr> </table>		ones	minus	tens	less than	hundreds	more than	thousands	multiply	mental	divide	mentally	product	add	calculate	sum of	digit	total	column addition	subtract	column subtraction	take away	estimate		inverse		operation
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			check
Show me... , And another ...	Convince me	What's the same? What's different? (Odd one out)	Always, sometimes, never
<p>"... two numbers that are easy to add ... two numbers that are hard to add ... two numbers that are easy to subtract ... two numbers that are hard to subtract ... two four digit numbers you can add ... a two digit number and one digit number you can multiply ... a three digit number and one digit number you can multiply</p>	<p>"... that 17×3 is the same as the sum of 10×3 and 7×3 ... that 14×6 will give a different answer to 16×4 ... that if I know that $468 / 4$ is 117, then I can check I am right by calculating 4×117</p>	<p>1310, 2250, 3460, 1325</p> <hr/> <p>45×5, 25×9, 15×15, 10×20</p> <p>45×9, 25×4, 15×7, 10×8</p>	<p>A four digit number add a four digit number equals a eight digit number A four digit number subtract a three digit number equals a one digit number A two digit number multiplied by a one number equals a two digit number A three digit number multiplied by a one number equals a three digit number Addition makes a number larger Subtraction makes a number smaller</p>
Misconceptions		Guidance	
<p>Addition/Subtraction Children sometimes struggle when adding numbers of different lengths to know what to do with a 'blank' in a column. As before, children struggle when adding and subtracting when they need to exchange a thousand/hundred/ten or vice versa - often they misrecord the remaining digit or fail to take account of any additional tens/hundreds etc gathered through addition. Children find examples where multiple exchanges must be made particularly</p>		<p>For addition, see guidance in Stages 2 and 3 about how to develop the formal methods in a conceptual way. Additionally, see the calculation policy for further guidance as well as the NCETM videos for exemplification! https://www.ncetm.org.uk/resources/40532</p> <p>Note that multiplication is introduced in Stage 3 and below in Unit 9 and Unit 13 (due to the focus on addition and subtraction in these year groups).</p>	



hard e.g. $4678 + 3945$ because the notation becomes unwieldy. Similarly subtractions such as $2304 - 1789$ cause issues because of the need to carry out a chain reaction of exchange. In these instances you may need to resort to equipment, even where the child does not need it for 'standard' calculations.

Multiplying

Children sometimes struggle to partition correctly when dividing up an array or using the grid method.

Weak times tables can lead to errors in larger calculations e.g. 40×7 is dependent on the knowledge of 4×7

When using the formal written method, children sometimes struggle to deal with situations where they need to exchange ones for a ten etc. and may forget to 'add in' any of these extra tens, hundreds etc

When teaching multiplication is important that children understand the two different representations i.e. 'lots of/'groups of' and 'scaling'. We often pay more attention to the former and hence problems involving the latter are not always even recognised as multiplication.

It is advised that you use a consistent meaning for a multiplication expression i.e. $a \times b$ means a multiplied by b and is represented by a objects (in a horizontal line) replicated in b rows. Thus the array for axb will be different for the array bxa (although they will contain the same number of dots). Arrays are recommended for representing multiplication and as a tool for starting to see how partitioning the number being multiplied (our a) helps you partition the array into two pieces.

It is critical that children can multiply single digits together (i.e. they know their times tables!) and that they can already multiply multiples of 10 by single digits e.g. 30×4 by comparing to 3×4 . This is in stage 3 but needs to be ensured before developing the Stage 4 content.

Try to move children from a concrete array of objects to a drawn array and on to a grid method representing the array before linking the grid method to the formal written method to see where that comes from.

When carrying out the formal written method, do the grid in parallel so that the exchange and regroup process can take place physically as the number is being transferred column in writing.

Activities

Show me what you know

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Stage 4		Unit 4: Exploring Shape	
Stage 3 support overview	Stage 4 core learning overview	Stage 5 extension overview	
<ul style="list-style-type: none"> ➤ identify horizontal and vertical lines and pairs of perpendicular and parallel lines ➤ recognise angles as a property of shape or a description of a turn identify right angles, recognise that two right angles make a half-turn, three make three quarters of a turn and four a complete turn; identify whether angles are greater than or less than a right angle 	<ul style="list-style-type: none"> ➤ identify lines of symmetry in 2-D shapes presented in different orientations ➤ identify acute and obtuse angles and compare and order angles up to two right angles by size ➤ compare and classify geometric shapes, including quadrilaterals and triangles, based on their properties and sizes 	<ul style="list-style-type: none"> ➤ know angles are measured in degrees: estimate and compare acute, obtuse and reflex angles ➤ use the properties of rectangles to deduce related facts and find missing lengths and angles ➤ distinguish between regular and irregular polygons based on reasoning about equal sides and angles. 	
Key learning steps		Key Vocabulary	
<ol style="list-style-type: none"> 1. I can identify lines of symmetry in 2D shapes 2. I can identify and describe all possible lines of symmetry in a 2D shape (horizontal, vertical, diagonal) 3. I can identify and distinguish between acute, right and obtuse angles 4. I can order and compare angles up to 180 degrees 5. I can recognise and describe the properties of 'famous' quadrilaterals 6. I can recognise and describe the properties of 'famous' triangles 7. I can sort 2d shapes using Carroll diagrams (one criterion) 8. I can sort 2D shapes by their properties using Venn diagrams (with 2 criteria) 		line of symmetry symmetrical angle acute right (angle) obtuse degrees greater than less than compare order properties Carroll diagram Venn diagram criterion, criteria sort, classify 2D dimension circle triangle equilateral triangle isosceles triangle	
		right-angled triangle scalene triangle quadrilateral rectangle square, oblong rhombus parallelogram kite arrowhead trapezium polygon horizontal vertical parallel perpendicular diagonal vertex	



Show me... , And another ...	Convince me	What's the same? What's different? (Odd one out)	Always, sometimes, never
<p>... a shape with exactly two line of symmetry</p> <p>... the lines of symmetry of this shape</p> <p>.. a shape with the same number of lines of symmetry as this shape</p> <p>... an angle that is less than this one/greater than this one</p> <p>... an obtuse angle, a right angle, an acute angle</p> <p>... an angle of about 30 degrees (EXT)</p> <p>... the quadrilateral family tree</p> <p>... a triangle that is equilateral</p>	<p>... a circle is not a polygon</p> <p>... an isosceles triangle has one line of symmetry</p> <p>... a square is a rectangle</p> <p>... a shape with four right angles exactly is a rectangle</p> <p>... all acute angles are smaller than all obtuse angles</p>	<p>... square, rectangle, oblong</p> <p>... parallelogram, rectangle, rhombus</p> <p>... trapezium, rectangle, circle</p> <p>... acute angle, right angle, obtuse angle</p> <p>... 32 degrees, 45 degrees, 90 degrees, 122 degrees</p>	<p>Right-angled triangles are scalene triangles.</p> <p>Squares are parallelograms</p> <p>Circles are polygons</p> <p>Pentagons have 5 lines of symmetry.</p> <p>Two acute angles together make an obtuse angle.</p>



... a polygon			
Misconceptions		Guidance	
<p>Children confuse the process of finding lines of symmetry with that of halving and quartering a shape. They may also be drawn particularly towards vertical and horizontal lines of symmetry, sometimes missing those at an angle. Similarly, some children 'see' diagonal lines of symmetry that are not there in reality because their eyes cannot process whether there is a 'match' with both sides of the picture at this angle.</p> <p>When describing angles, some children interchange acute and obtuse - they also don't realise that right angles are part of this progression i.e. your angle is either acute OR right OR obtuse if it is less than 180 degrees. Initially children may find it hard to attach the number of degrees to these words</p>		<p>When teaching symmetry, use mirrors and tracing paper for finding lines of symmetry as well as practical folding activities to demonstrate symmetry. It is easier to both see and to test out symmetry for the human brain if the mirror line is vertical (because of our eye formation) so encourage children to turn the paper or object so they can see it in this way to make their judgments. Look at more complex shapes with diagonal lines of symmetry if possible - include things like hexagons which are harder to visualise.</p> <p>Angle description - estimation isn't strictly necessary at this stage but it is advantageous. Try to get children to estimate how many degrees their angle is based on the knowledge that a right angle is 90. Number facts based around 90 will help this process.</p> <p>The section on properties of shapes is rich and complex - here are a few pointers:</p> <ul style="list-style-type: none"> - encourage children to see shapes as families rather than as individuals e.g. there is a polygon family, within which there is a triangle family, a quadrilateral family, a pentagon family and so on. These mini-families break down themselves e.g. quadrilaterals may be parallelograms (and then either rhombuses or rectangles or neither) and so on. - focus on mathematical language so that children are using words like sides, vertices, symmetry, parallel, perpendicular, right angles and so on - use lots of sorting activities practically initially to help develop the 'testing against a criterion' behaviour and the thoroughness needed - bring in representations of sorting later e.g. Carroll diagrams and Venn diagrams. <p>Throughout all of this use tricks with the language e.g. Acute = cute = small. Make links to word origins, root words and prefixes (quad = four, tri = three).</p>	
Activities		Show me what you know	



NUFFIELD AMP: Symmetry
NRICH: Let's Reflect *
NRICH: National Flags *
NRICH: Stringy Quads **

NRICH: Nine-pin Triangles ***
NRICH: Cut it Out ***

BOWLAND assessments: Three of a Kind

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Stage 4		Unit 5: Generalising Arithmetic															
Stage 3 support overview	Stage 4 core learning overview	Stage 5 extension overview															
<ul style="list-style-type: none"> ➤ "add and subtract numbers mentally, including: ➤ - a three-digit number and ones ➤ - a three-digit number and tens ➤ - a three-digit number and hundreds" ➤ add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction ➤ solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction 	<ul style="list-style-type: none"> ➤ add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate ➤ multiply two-digit and three-digit numbers by a one-digit number using formal written layout ➤ use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers ➤ solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why 	<ul style="list-style-type: none"> ➤ add and subtract numbers mentally with increasingly large numbers ➤ add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction) ➤ multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers ➤ divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context ➤ multiply and divide whole numbers and those involving decimals by 10, 100 and 1000 ➤ solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why 															
Key learning steps		Key Vocabulary															
<ol style="list-style-type: none"> 1. I can add two four digit numbers using a column method 2. I can subtract two four digit numbers using a column method 3. I can multiply a two digit number by a one digit number using a formal written method 4. I can multiply a three digit number by a one digit number using a formal written method 5. I can use place value and number facts to multiply mentally including multiplying by 0 and 1 6. I can use place value and number facts to divide mentally including dividing by 1 7. I can multiply three numbers together mentally 		<table style="width: 100%; border: none;"> <tr> <td style="width: 50%;">ones</td> <td style="width: 50%;">multiply</td> </tr> <tr> <td>tens</td> <td>divide</td> </tr> <tr> <td>hundreds</td> <td>product</td> </tr> <tr> <td>thousands</td> <td>calculate</td> </tr> <tr> <td>mental</td> <td>digit</td> </tr> <tr> <td>mentally</td> <td>column addition</td> </tr> <tr> <td>add</td> <td>column subtraction</td> </tr> </table>		ones	multiply	tens	divide	hundreds	product	thousands	calculate	mental	digit	mentally	column addition	add	column subtraction
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tens	divide																
hundreds	product																
thousands	calculate																
mental	digit																
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add	column subtraction																



8. I can solve 2 step addition and subtraction problems choosing the correct operation and using the most appropriate methods		altogether sum of total subtract take away minus less than more than	inverse operation check place value partition carry borrow
Show me... , And another ...	Convince me	What's the same? What's different? (Odd one out)	Always, sometimes, never
<p>... how you can calculate 46×7 using</p> <ul style="list-style-type: none"> - a grid method - partitioning - a column method - using near multiples <p>... how you can calculate 0.4×7</p> <p>...how you can calculate $8.4 / 7$</p> <p>... how you would find the factor of 360 that makes a pair with 72</p>	<p>"... A boy worked out how many 19p stamps you can buy for £5. His answer was 25. Do you think he was right or wrong? Why?</p> <p>... That multiplying by 24 is the same as multiplying by 12 and 2</p> <p>... that dividing by 24 is the same as dividing by 12 and 2</p> <p>... that this calculation is right $435 \times 6 = 2610$</p>	<p>"5×7; 0.5×7; 5×0.7; 0.5×0.7</p> <p>$18 \div 2$, $18 \div 3$, $18 \div 4$, $18 \div 5$</p>	<p>When you add, you can work left to right or right to left, the order doesn't matter</p> <p>When you multiply, you can do the tens first or the ones first, the order doesn't matter.</p>
Misconceptions		Guidance	
<p>"Children can sometimes be found failing to carry or borrow when they need to as well as carrying or borrowing when it is not necessary</p> <p>Once numbers have been carried, some children will forget to include the tens or hundreds they have carried</p> <p>In subtraction, children will sometimes borrow from the wrong column. Additionally, they may place the smallest number at the top of the calculation when using column subtraction.</p>		<p>Develop secure place value as a precursor to effective addition and subtraction</p> <p>At this level, use place value counters to help understand this process initially, developing a practical and written solution together before reverting to written only. You can use the counters to represent any column you wish, e.g. tenths or thousands as required.</p> <p>Use rounding to estimate calculations to support the checking of answers.</p>	



<p>Children sometimes partition incorrectly when using grid method to multiply In long multiplication it is common for children to multiply by the wrong digits or to miss out one of the multiplications.</p> <p>Children sometimes use the incorrect operation when checking</p> <p>They sometimes make errors when multiplying by 1 or 0, saying that $3 \times 0 = 3$, for example.</p> <p>In solving problems, children may complete the first step of a problem and use it as the answer without going any further</p>	<p>Distinguish between calculations that require carrying or borrowing and those that don't</p> <p>Ensure children understand partitioning before using a grid method to multiply - build this up from arrays so children can 'see' the array that sits behind the grid method.</p> <p>Use a strategy of combining array and grid to start with before moving to just grid ultimately.</p> <p>Recap on multiplying by 10,100 and multiples of 10 and 100 before multiplying 2 and 3 digit numbers</p> <p>Draw attention to language used within addition and subtraction problems</p>
<p>Activities</p>	<p>Show me what you know</p>
<p>NRICH: Trebling *</p> <p>NRICH: All the Digits **</p> <p>NRICH: The Puzzling Sweet Shop **</p> <p>NRICH: Money Bags **</p> <p>NRICH: Amy's Dominoes **</p> <p>NRICH: Escape from the Castle **</p> <p>NRICH: Fifteen Cards *</p> <p>NRICH: Sealed Solution **</p> <p>NRICH: Roll These Dice **</p>	<p>Click here to access files in Google drive</p>



Stage 4		Unit 6: Reasoning with Measures	
Stage 3 support overview	Stage 4 core learning overview	Stage 5 extension overview	
<ul style="list-style-type: none"> ➤ add and subtract amounts of money to give change, using both £ and p in practical contexts ➤ measure the perimeter of simple 2-D shapes 	<ul style="list-style-type: none"> ➤ estimate, compare and calculate different measures, including money in pounds and pence ➤ measure and calculate the perimeter of a rectilinear figure (including squares) in centimetres and metres ➤ find the area of rectilinear shapes by counting squares 	<ul style="list-style-type: none"> ➤ measure and calculate the perimeter of composite rectilinear shapes in centimetres and metres ➤ calculate and compare the area of rectangles (including squares), and including using standard units, square centimetres (cm²) and square metres(m²) and estimate the area of irregular shapes ➤ "estimate volume [for example, using 1 cm³ blocks to build cuboids (including cubes)] and capacity [for example, using water]" 	
Key learning steps		Key Vocabulary	
<ol style="list-style-type: none"> 1. I know the approx lengths/weights of some familiar objects that are eg 1, 10, 20, 50, 100 cm/g, 1, 10, 20, 50, 100m/kg and can use this knowledge to estimate lengths and weights. 2. I can use multiplication to calculate the cost of buying several of the same thing and combine this with addition and subtraction to get total costs and change 3. I can use division to calculate shares of a bill or how many of the same thing can be bought for a given amount 4. I can estimate the cost of several items or the number that can be bought with a given amount by rounding prices to easier amounts 5. I can measure or draw lines accurately in mm, cm and m, using a ruler or metre stick, using composite units or decimals (2 cm 3 mm = 2.3 cm) 6. "I can measure the perimeter of squares and rectangles using mm, cm and m. 7. I can solve problems linking perimeters to costs" 8. I can find the area of a square or rectangle by counting the cm squares it takes to fill the shape, and I can work out the area of a right angled triangle by treating it as half a rectangle 9. I can explain why the area of a rectangle is length x width by referring to counting squares in rows or columns 		accurate; add; angle; approximate; area; calculate; composite; decimal; divide; estimate; gram, g; kilogram, kg; metre, m; metre-rule; millimetre, mm; multiply; pair of compasses; price; right-angle; round; set-square; share; triangle	



Show me... , And another ...		Convince me		What's the same? What's different? (Odd one out)		Always, sometimes, never	
Something in the room that is 20cm long. (3m, 4mm, etc). Something that weighs about 100g. (500g, 10g etc)		That the area of a 4cm by 5cm rectangle is $4 \times 5 = 20\text{cm}^2$. (Not allowed to quote formula) That dropping a 1g weight on my head won't hurt.		A 6cm by 6cm square and a 7cm by 5cm rectangle. Same perimeter, different areas; Same angles, different sides.		A square is a rectangle. A triangle has less area than a rectangle (half a big rectangle can be more than a small rectangle)	
Misconceptions				Guidance			
confusing units of weight or length; using repeated + instead of x; not knowing x tables; lack of sharing and grouping concepts of division; lack of relevant arithmetic skills; not including all sides in perimeter; difficulty in measuring objects longer than the ruler; drawing rectangles of wrong size; knowing $A = l \times w$ but not why; lack of concept of area as number of unit squares needed to fill a space;				The first lesson is aiming to develop a basis of experience to support estimation. Most children find estimation hard and resort to wild guesses or measuring first then 'estimating' (often revealed by estimates that are too good to be true). Money work is now linking to multiplication and division. This needs to be differentiated depending on number skills. It is important to link division with concrete activities, involving both interpretations: sharing and grouping. Measuring skills are extended to develop accurate measurements. Early work on area should not start with formulas. The key concept is counting how many squares are needed to fill the space. The triangle is dealt with by realising it is half of a rectangle (NO formula yet).			
Activities				Show me what you know			
NRICH: Torn Shapes *				Click here to access files in Google drive			
NRICH: Discuss and Choose *							



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Stage 4		
Unit 7: Discovering Equivalence		
Stage 3 support overview	Stage 4 core learning overview	Stage 5 extension overview
<ul style="list-style-type: none"> ➤ recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators ➤ recognise and use fractions as numbers: unit fractions and non-unit fractions with small denominators ➤ compare and order unit fractions, and fractions with the same denominators ➤ recognise and show, using diagrams, equivalent fractions with small denominators identify, represent and estimate numbers using different representations 	<ul style="list-style-type: none"> ➤ recognise and show, using diagrams, families of common equivalent fractions ➤ count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10 ➤ recognise and write decimal equivalents of any number of tenths or hundredths ➤ recognise and write decimal equivalents to $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ 	<ul style="list-style-type: none"> ➤ recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number [for example, $\frac{2}{5} + \frac{4}{5} = \frac{6}{5} = 1 \frac{1}{5}$ ➤ compare and order fractions whose denominators are all multiples of the same number ➤ identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths ➤ count up and down in hundredths; recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten ➤ read and write decimal numbers as fractions [for example, $0.71 = \frac{71}{100}$] ➤ recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents ➤ recognise the per cent symbol (%) and understand that per cent relates to 'number of parts per hundred' ➤ write percentages as a fraction with denominator 100, and as a decimal ➤ solve problems which require knowing percentage and decimal equivalents of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{4}{5}$ and those fractions with a denominator of a multiple of 10 or 25.



Key learning steps		Key Vocabulary	
<ol style="list-style-type: none"> 1. I can recognise representations of equivalent fractions 2. I can draw diagrams to show equivalent fractions 3. I can count up and down in tenths 4. I can group sets of objects into tenths by splitting them into 10 equal groups 5. I can find a tenth of a whole number by dividing it by 10 (including single digits) 6. I can write tenths as decimals by using their place value headings 7. I can write hundredths as decimals by using their place value headings 8. I can write the decimal equivalents of $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{3}{4}$ 		fraction decimal tenth hundredth equivalent family pattern divide place value representation decimal equivalent	
Show me... , And another ...	Convince me	What's the same? What's different? (Odd one out)	Always, sometimes, never
<p>... where $\frac{1}{10}$ sits on the number line</p> <p>... what comes next $\frac{7}{10}$, $\frac{8}{10}$, $\frac{9}{10}$, ...</p> <p>... how you can show $\frac{3}{10}$ of this shape? of this number? on the number line? as a decimal?</p> <p>... an equivalent fraction to $\frac{2}{10}$</p> <p>... what comes next in this pattern: $\frac{3}{10}$, $\frac{6}{20}$, $\frac{9}{30}$, $\frac{12}{40}$,</p>	<p>... that finding $\frac{1}{10}$ of a quantity is the same as dividing by 10</p> <p>... that $\frac{20}{100}$ is equivalent to two tenths (in more than one way!)</p> <p>... that $\frac{1}{4} = 0.25$</p> <p>... that $\frac{1}{10}$ of 14 = 1.4</p>	<p>$\frac{7}{10}$, 0.7, $\frac{7}{100}$, $\frac{14}{20}$</p> <p>$\frac{1}{4}$, $\frac{1}{2}$, 0.5, 0.25, $\frac{3}{4}$, $\frac{2}{4}$, 0.75</p> <p>tenth, $\frac{1}{10}$, 0.1, \div 10, $\frac{10}{100}$</p>	<p>... you find a tenth of a number by removing its final zero</p> <p>... when you write a fraction with a denominator of 100 as a decimal, the decimal will have two decimal places</p>
Misconceptions		Guidance	
Children find it hard to recognise equivalent fractions by the numbers themselves (although some will see a pattern) and so need a visual		Make use of a visual aid when counting or chanting to represent the tenths using both a number line as well as a proportion (e.g. circles cut into ten equal	



<p>representation to spot equivalent fractions</p> <p>Pupils may also confuse 'tens' and 'tenths' and similarly 'hundreds' and 'hundredths'.</p> <p>Many children struggle to divide numbers by 10 as this is not yet secure - look out for children who just try to 'take off a zero' who therefore struggle when there is no zero to remove</p> <p>When counting in tenths, some children may find it hard to use the whole numbers when a multiple of ten is reached.</p>	<p>pieces). Match this with the fraction symbol and the place value headings to reinforce the equivalence of all of these representations of 'tenths'. Big Maths have some ppts for this that could be used as a starting point http://www.andrelleducation.com/product/big-maths-classroom-activities/ This will gradually filter into the children's awareness so that they can then work with and change between representation as they need to - this is critical mathematical skill.</p> <p>This is also a good opportunity to get out your counting stick, both to support counting in tenths and the concept of fractions.</p> <p>When teaching children about dividing by 10, it is useful to begin with multiples of 10. However, they also need to be able to apply this to non-multiples, hence the emphasis on tenths and the equivalence of decimals with these. The logic is that, for example, a tenth of 14 is the same as 14 divided by 10, which is one 10 and four 1s divided by 10, or one 1 and 4 tenths, which is equivalent to 1.4. You will need to make use of lots of models and images showing place value headings after the decimal point to achieve this logical flow! The shortcut of simply 'removing a 0' or 'sticking in a decimal point' should be avoided until the conceptual route is established - if children want to use this shortcut too early before understanding is strong, throw in some deliberately tricky questions to catch them out to show them why they cannot just use their method. E.g. find a tenth of 34.0</p>
<p>Activities</p> <p>NRICH: Matching Fractions (Pelmanism) http://nrich.maths.org/8283/note</p> <p>Fractions ITP (Nat Strat) http://www.taw.org.uk/lic/itp/fractions.html</p> <p>Fraction manipulatives - exploring equivalence http://donnayoung.org/math/fraction.htm</p> <p>Fraction models and support questions - http://www.annery-kiln.eu/gaps-misconceptions/all-images.html</p>	<p>Show me what you know</p> <p>Click here to access files in Google drive</p>





Stage 4		Unit 8: Investigating Statistics																			
Stage 3 support overview		Stage 4 core learning overview	Stage 5 extension overview																		
<ul style="list-style-type: none"> interpret and present data using bar charts, pictograms and tables solve one-step and two-step questions [for example, 'How many more?' and 'How many fewer?'] using information presented in scaled bar charts and pictograms and tables 		<ul style="list-style-type: none"> interpret and present discrete and continuous data using appropriate graphical methods, including bar charts and time graphs solve comparison, sum and difference problems using information presented in bar charts, pictograms, tables and other graphs 	<ul style="list-style-type: none"> solve comparison, sum and difference problems using information presented in a line graph complete, read and interpret information in tables, including timetables 																		
Key learning steps		Key Vocabulary																			
<ol style="list-style-type: none"> I can recognise discrete and continuous data and come up with appropriate categories for it I can record discrete or continuous data in a frequency table I can construct a bar chart or frequency diagram to represent discrete or continuous data correctly. I can interpret a bar chart, including reading several different values to answer a more complex question in the context of the original problem. I can explain and understand the limitations where data is grouped. I can construct line graphs and time graphs correctly. I can interpret a line graph and a time graph I can select appropriate charts and read a range of charts to solve comparison problems. 		<table border="0"> <tr> <td>"data</td> <td>most</td> </tr> <tr> <td>discrete</td> <td>least</td> </tr> <tr> <td>continuous</td> <td>sum</td> </tr> <tr> <td>bar chart</td> <td>difference</td> </tr> <tr> <td>class intervals</td> <td>compare</td> </tr> <tr> <td>frequency diagram</td> <td>construct</td> </tr> <tr> <td>line graph</td> <td>interpret</td> </tr> <tr> <td>time graphs</td> <td></td> </tr> <tr> <td>trend</td> <td></td> </tr> </table>		"data	most	discrete	least	continuous	sum	bar chart	difference	class intervals	compare	frequency diagram	construct	line graph	interpret	time graphs		trend	
"data	most																				
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time graphs																					
trend																					
Show me... , And another ...	Convince me	What's the same? What's different? (Odd one out)	Always, sometimes, never																		
<ul style="list-style-type: none"> ... some data that is discrete/categoric/continuous ... a sketch of a bar chart ... a sketch of a frequency diagram ... how you can sort this data out into classes/categories ... which chart would be best to display: 	<ul style="list-style-type: none"> ... that these are equal class intervals ... that a line graph is the best to use for this data (temperature each month) 	<ul style="list-style-type: none"> ... bar chart and frequency diagram ... discrete and continuous data ... bar chart and line graph 	<ul style="list-style-type: none"> ... when drawing a bar chart you want to make the step size as small as possible ... bar charts have bars that do not touch ... all data is discrete once we measure it 																		



<ul style="list-style-type: none"> - A person's height from age 0 to age 20. - A person's pulse rate during the data. - A class' favourite colour. - The pupils' favourite music from a year group at school. - The sales of ice creams at a shop over a month in July. - Votes for all the celebrities in a tv talent contest for one show. - Votes for one celebrity in a tv talent contest for a series of shows. 			
Misconceptions		Guidance	
<p>Children find it hard to see the difference between discrete and continuous data - often because the way we measure and record continuous data makes it sort of discrete when we write it down e.g. heights are continuous because they can take any value but if we are measuring the nearest cm then they can't take ANY value in our study and so they are to some extent now discrete!</p> <p>Children forget that bar charts should have gaps between them (as the data is discrete) and frequency diagrams have bars that touch as the data is continuous.</p> <p>Children use bars for line graphs and vice versa</p>		<p>Be aware that a bar chart represents discrete data and as such the bars should not touch each other. When the data is continuous, a frequency diagram (later histogram) should used and the bars will touch because the categories connect. At this level, there will mostly be bar charts as any continuous data takes the form of a time series that can be plotted using a line graph instead.</p> <p>Make sure children have a chance to explore what type of graph would be appropriate for a specific set of data and question. They need to understand what a line graph gives you that a bar chart/frequency diagram doesn't and know the sort of situations where you would naturally use one.</p> <p>It can be good to get children to come up with their own axes and scales as a challenge - and even to compare two data set using bar charts.</p>	
Activities		Show me what you know	
		<p>Click here to access files in Google drive</p>	



Stage 4		Unit 9: Solving Number Problems	
Stage 3 support overview	Stage 4 core learning overview	Stage 5 extension overview	
<ul style="list-style-type: none"> ➤ write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods ➤ solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects 	<ul style="list-style-type: none"> ➤ find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths ➤ "use place value, known and derived facts to multiply and divide mentally, including: <ul style="list-style-type: none"> ➤ multiplying by 0 and 1; ➤ dividing by 1; ➤ multiplying together three numbers" ➤ multiply two-digit and three-digit numbers by a one-digit number using formal written layout ➤ solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects ➤ solve simple measure and money problems involving fractions and decimals to two decimal places 	<ul style="list-style-type: none"> ➤ multiply and divide whole numbers and those involving decimals by 10, 100 and 1000 ➤ multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers ➤ divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context ➤ solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes ➤ solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign ➤ solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates ➤ solve problems involving number up to three decimal places 	
Key learning steps		Key Vocabulary	



<ol style="list-style-type: none"> 1. I can divide a 1d or 2d number by 10. 2. I can divide a 1d or 2d number by 100. 3. I can multiply any 2d by a 1d using the distributive law. 4. I can use place value to multiply and divide by 0 and 1 5. I can use an efficient written method to multiply TU x U and HTU x U 6. I can solve more complex correspondence problems where n objects are connected to m objects. 7. I can solve measure and money problems involving multiplication and division. 8. I can solve measure and money problems that involve fractions and decimals 		addition multiplication division divide place value digits partition 1s/ones tenths	hundredths distributive law solve problem represent array grid scaling bar model
Show me... , And another ...	Convince me	What's the same? What's different? (Odd one out)	Always, sometimes, never
<p>... $24 \div 10$... $24 \div 100$... $124 \div 10$... $124 \div 100$... $240 \div 10$... $240 \div 100$</p> <p>... the single calculation that is equivalent to $20 \times 4 + 5 \times 4$</p> <p>... how you could represent 73×6 using an array? a grid method? two calculations?</p> <p>... how you can represent this problem: Jodie has 8 crates containing 24 bottles. How many bottles does she have in total?</p>	<p>... that $230 \div 100 = 2.3$</p> <p>... that $65 \times 100 = 6500$</p> <p>... that $15 \times 9 = 135$</p> <p>...</p>	<p>... $40 \times 7 + 2 \times 7$, 47×2, 42×7 and $40 \times 2 + 7 \times 2$</p> <p>... the number that is 4 times bigger than 23, 23×4, 4 lots of 23, the product of 4 and 23</p> <p>... grid, array, partitioned calculation, column method, bar model</p>	<p>... when you divide a number by 10, you remove one zero from the end</p> <p>... when you divide a number by 100, you will end up with a number with hundredths in</p> <p>...</p>
Misconceptions		Guidance	
Children think that dividing by 10 means taking the zero off the end and		Note: this unit does not directly cover the greyed out objectives (which have	



multiplying by 10 means adding it. They do not relate multiplying and dividing to the place value and unitisation of a number e.g. 24 x 10 is 2 tens and 4 ones multiplied by 10 which will be 2 hundreds and 4 tens or 240. This can lead to errors where a decimal point is needed and not used or vice versa.

Some children still experience confusion over tenths and tens, hundreds and hundredths - they may not correctly label columns as a sign of this.

When carrying out more complex multiplications, some children will fail to realise that multiplication is commutative and struggle to use the times tables that they know to tackle a related question.

Children may struggle to represent scaling and correspondence problems visually (because they don't conform to the 'lots of' imagery that some children focus on for multiplication)

Children find it hard to separate how you can 'make' a number by both ADDING and MULTIPLYING - they may lean towards additive relationships more than multiplicative e.g. they may not have understanding of how 24 can be made of 10 and 14 as well as 20 and 4 (and other examples).

been studied already) - however, if your children are not secure on these, you need to address this before moving them on to the more complex skills contained in this unit.

Mult/Div by 10 and 100

Make use of physical/active examples of digits moving when multiplying or dividing by 10, 100 etc. to show how the different place value units relate. Get children up and moving so they see the digits move but not the decimal point. Use place value cards so children can see that 1 digit in the thousands column also has lots of zeros after it.

Also ensure that children can 'read' the answer to these questions e.g. 0.24 is 2 tenths and 4 hundredths (or you could say 24 hundredths, which is often missed out).

Use a hundred-grid to show why 2 tenths is the same as 20 hundredths etc.

By this stage children should be working on or confident with ALL time tables - therefore, when solidifying multiplication processes, ensure they encounter numbers from across these times tables. Refer to the calculation policy for more detail on the progression of these concepts.

You need to expose the children to a lot of different problem solving using multiplication (and addition and division) in this unit - it should be at least half!!! Try to encourage children to represent the problem first to decide which calculations to do and then to carry these out below. The Bar Model is really useful as a consistent way of representing problems (be they word problems, real life problems or more abstract problems). You may need to model this first to help children see how to use it - see the guidance at the NCETM on the bar model here <https://www.ncetm.org.uk/resources/44568>

Activities

Show me what you know

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