Stage 5	Unit 1: Pattern Sniffing	
Stage 4 support overview	**Stage 5 core learning overview**	**Stage 6 extension overview**
➢ count in multiples of 6, 7, 9, 25 and 1000 ➢ find 1000 more or less than a given number ➢ recall multiplication and division facts for multiplication tables up to 12 × 12 ➢ recognise and use factor pairs and commutativity in mental calculations	➢ count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000 ➢ recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3}) ➢ multiply and divide numbers mentally drawing upon known facts ➢ identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers ➢ establish whether a number up to 100 is prime and recall prime numbers up to 19	➢ **generate and describe linear number sequences** ➢ **identify common factors, common multiples and prime numbers**

Key learning steps	Key Vocabulary
1. I can count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000 2. I can count forwards and backwards with positive and negative whole numbers, including through zero 3. I can recognise and use square numbers and the notation for squared (^{2}) 4. I can recognise and use cube numbers and the notation for cubed (^{3}) 5. I can multiply and divide numbers, mentally drawing upon known facts 6. I can identify multiples and factors of numbers 7. I can find all factor pairs of a number and common factors of two numbers 8. I can use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers; I can establish whether number up to 100 is prime and recall prime numbers up to 19	Powers (of 10) square number cube number negative number positive number prime composite number factor multiple

Show me... , And another ...	Convince me	What's the same? What's different? (Odd one out)	Always, sometimes, never
10 more than xxx, 100 more than, 1000 more than ,,,, 10 less than xxx, 100 less than, 1000 less than ,,,, A square number A cube number	...4090 + 10 ≠ 5000 ...4 - 6 = -2 ...-3 + 7 = 4 ...8 is a factor of 56 ...90 is a mutiple of 3 ...2 is a prime number	1, 3, 7, 11 2, 5, 10, 20	A number has an even number of factors A number has an even number of multiples Pick a number, multiply by 6, add 1. The answer is a prime number.

A multiple of 5, 6, xxxx A factor of 60, xxx A common factor of 24 and 40 A prime number < 19 A composite number	...1 is not a prime number		Prime numbers are odd Prime numbers can be a multiple of 4
Misconceptions		**Guidance**	
When counting in powers of 10, pupils struggle when bridging 10, 100 etc e.g. they think that 997 + 100 = 1197 and forget about 1097. Children forget that 1 is a factor of any number and that the number itself is both a factor and a multiple of itself. Children also interchange the meanings of factor and multiple frequently. Pupils think 1 is a prime number Pupils think 2 is not a prime number		Ensure times table recall is developing to help children 'spot' factors of numbers more easily. Develop other facts from initial facts e.g. if you know that 4 is a factor of 16 then you also know that .... At this stage find common factors and multiples purely by listing and comparing, although you can encourage children to find efficient ways to do this.	
Activities		**Show me what you know**	
Recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3}) NRICH: Up and Down Staircases * NRICH: One Wasn't Square ** NRICH: Cycling Squares ** Identify multiples and factors, including all factor pairs of a number, and common factors of two numbers NRICH: Sweets in a Box * NRICH: Which Is Quicker? * NRICH: Multiplication Squares * NRICH: Flashing Lights * NRICH: Abundant Numbers *		Click here to access files in Google drive	

Stage 5	Unit 2: Investigating Number Systems	
Stage 4 support overview	**Stage 5 core learning overview**	**Stage 6 extension overview**
➢ read Roman numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of zero and place value ➢ recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones) ➢ identify, represent and estimate numbers using different representations ➢ solve number and practical problems that involve all of the above and with increasingly large positive numbers ➢ round any number to the nearest 10, 100 or 1000 ➢ round decimals with one decimal place to the nearest whole number ➢ order and compare numbers beyond 1000 ➢ compare numbers with the same number of decimal places up to two decimal places	➢ read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit ➢ read Roman numerals to 1000 (M) and recognise years written in Roman numerals ➢ read, write and interpret negative numbers in context ➢ solve number problems and practical problems that involve all of the above ➢ round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000 ➢ round decimals with two decimal places to the nearest whole number and to one decimal place ➢ read, write, order and compare numbers with up to three decimal places	➢ **read, write, order and compare numbers up to 10 000 000 and determine the value of each digit** ➢ **use negative numbers in context, and calculate intervals across zero** ➢ **solve number and practical problems that involve all of the above** ➢ **round any whole number to a required degree of accuracy** ➢ **identify the value of each digit in numbers given to three decimal place**

Key learning steps	Key Vocabulary	
1. I can read and write numbers beyond 100000 2. I can read Roman Numerals to 1000 and recognise years written in Roman Numerals. 3. I can position and estimate positive and negative numbers on a number line or other representation. 4. I can round any decimal with two decimal places to the nearest integer or 1 decimal place. 5. I can use and apply above number knowledge to solve number problems. 6. I can round any number to the nearest 10, 100, 1000, 10000 or 1000000 7. I can compare numbers with up to three decimal places, using the signs <, > (and =) to show this comparison. 8. I can order decimals with up to 3 decimal places	Hundreds Thousands Ten Thousands Millions Place Value Order Compare Numerals Position	Estimate Positive Negative Round Rounding Nearest Decimals Decimal place Integer

Show me... , And another ...	Convince me	What's the same? What's different? (Odd one out)	Always, sometimes, never
... the number 3 million four hundred	.... that 0.35 is greater than 0.035?	72.344 and 72.346	3.5 is closer to 4 than it is to 3

and fifty-seven thousand, six hundred and fifty-four in symbols ... the number 2, 045, 678 in words ... where 345, 678 would be on this number strip that goes from - 0-1000,000 - 300,000 - 400,000 - 345,000 - 350,000 - 345,000 - 346,000 ... a number that rounds to 546,000 when rounded to the nearest 1000 ... a number that rounds to 567,800 when rounded to the nearest 100 ... a number that rounds to 2.6 when rounded to 1 decimal place ... a possible value for ? in 5.4 < ? < 5.51 .. a number between 0.12 and 0.17. Which of the two numbers is it closer to? How do you know? ... how you order these numbers 7.765, 7.675, 6.765, 7.756, 6.776	...that these numbers are in ascending order: 3.41, 3.419, 3.5, 3.507, 3.52 ... that both 567,501 and 568499 round to 568000 to the nearest thousand ...why might it not be possible to identify the first three places in a long jump competition if measurements were taken in metres to one decimal place	-5, -50, 50, 5 -6, -5, -2, 4 5.67, 5.69, 5.73, 5.64	–36 is greater than –34 0 is greater than 9, so 0.10 is greater than 0.9 There is only one pair of numbers with a sum of 3 and difference of 11

Misconceptions	**Guidance**
Pupils may mispronounce or misread or miswrite larger numbers involving ten thousands and millions e.g. Three hundred and twenty-seven thousand and four hundred and fifty-six Similarly they may do this with decimals, saying 'two point two hundred and forty-seven' rather than 'two point two four seven' for 2.247 Children struggle with the different concepts of the magnitude of a number and the sign of a number e.g. they think that e.g. -6 is greater than 3. It is important that they understand that 'greater' means 'higher up the number line' Children do not always fully understand the role of 0 as a place holder and hence struggle to read or write numbers like 20,045	Ensure children read the numbers as well as write them down to make sure they fully understand the place value within each number and are not just following a process. Use real life examples of where and when Roman numerals are used. Explore the history of when and why they were used. Possible topic link. Use number lines when exploring negative numbers to show where they place in relation to positive numbers and decimals. Ensure children understand the role of 0 as a place holder e.g. 0.07 differs from 0.70, and 0.7 is equal to 0.70

Children 'miss out' ten thousands frequently, jumping straight from thousands to millions in terms of column headings for place value. When rounding, children sometimes want to round up in every case and they do not look carefully at the next number to decide whether to leave the stem alone or whether to round the final digit up. They also sometimes fail to check only the next digit, instead looking at every digit from the end of the number and rounding along in a 'chain reaction'	
Activities	**Show me what you know**
NRICH: Tug Harder! * STANDARDS UNIT: N8 Using Directed Numbers in Context	Click here to access files in Google drive

Stage 5	Unit 3: Solving Calculation Problems	
Stage 4 support overview	**Stage 5 core learning overview**	**Stage 6 extension overview**
➢ add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate ➢ multiply two-digit and three-digit numbers by a one-digit number using formal written layout ➢ estimate and use inverse operations to check answers to a calculation	➢ add and subtract numbers mentally with increasingly large numbers ➢ add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction) ➢ multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers ➢ divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context ➢ use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy	➢ **perform mental calculations, including with mixed operations and large numbers** ➢ **multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication** ➢ **divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context** ➢ **use their knowledge of the order of operations to carry out calculations involving the four operations** ➢ **solve problems involving addition, subtraction, multiplication and division** ➢ **use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy** ➢ **use simple formulae** ➢ **"recognise when it is possible to use formula for area and volume of shapes (Measurement)"**
Key learning steps		**Key Vocabulary**

1. I can add and subtract numbers involving three and four digits mentally 2. I can add and subtract numbers with more than four digits using a columnar method 3. I can multiply a four digit number by a one digit number using long multiplication. 4. I can multiply a four digit number by a two digit number using an informal method or representation. 5. I can multiply a four digit number by a two digit number using long multiplication 6. I can divide a four digit number by a one digit number and find remainders using a formal written method of short division 7. I can interpret a remainder in the context of a problem. 8. I can use rounding to check answers to calculations and problems	ones tens hundreds thousands mental mentally add sum of total subtract take away minus less than more than multiply multiple	divide round rounding product calculate digit column addition column subtraction estimate inverse operation check remainders lots of groups of nearest

Show me... , And another ...	Convince me	What's the same? What's different? (Odd one out)	Always, sometimes, never
... two numbers that are easy to add ... two numbers that are hard to add ... two numbers that are easy to subtract ... two numbers that are hard to subtract ... two numbers that are easy to multiply ... two numbers that are hard to multiply ... two numbers that are easy to divide ... two numbers that are hard to divide ... a division with a remainder ... a division without a remainder	... that 453 x 28 is the same as 453 x 20 + 453 x 8 which is the same as 400 x 28 + 50 x 28 + 3 x 28 ... that 715 x 79 cannot equal 42075 ... that I will need 8 coaches to take 375 children on a trip using coaches that seat 53 children each.	63, 7, 441, 371 125/5, 98/4, 145/9, 126/6 456x4, 312x20, 458x27, 689x50	A four digit number multiplied by a two number equals an eight digit number Long multiplication is needed to multiply four digit numbers by two digit numbers A calculation involving division will have a remainder Division is the inverse of multiplication Addition makes a number larger Subtraction makes a number smaller

Misconceptions	Guidance

Addition/Subtraction With numbers with 4 or more digits, there can be an issue in reading and writing these down correctly e.g. a number such as two million, thirty-two thousand, four hundred and five may be miswritten as 2300405 rather than 2030405 as it should be. It is the 0s that cause most problems. As before, children struggle when adding and subtracting when they need to exchange a million/hundred thousand/ten thousand/thousand/hundred/ten or vice versa - often they misrecord the remaining digit or fail to take account of any additional tens/hundreds etc gathered through addition. Children find examples where multiple exchanges must be made particularly hard e.g. 24678 + 13945 because the notation becomes unwieldy. Similarly subtractions such as 23004 - 17689 cause issues because of the need to carry out a chain reaction of exchange. In these instances you may need to resort to equipment, even where the child does not need it for 'standard' calculations. When performing columnar addition, children sometimes forget to include the tens, hundreds or thousands they have carried. Multiplying When doing long multiplication, children struggle to record multiple multiplications in one answer row e.g. when working out 432 x 8 you record 2x8, 30 x8 and then 400 x 8 in the same row of the answer. They may become confused with examples where there is an exchange (or a number is carried over). Poor understanding of how to multiply multiples of 10, 100 etc may lead to large errors in calculations using the grid method e.g. children often mistakenly say that 40 x 50 = 200 using the 'number of zeroes in questions = number of zeroes in answer' rule incorrectly. Weak times tables will cause errors in all methods for multiplication - this is a priority for ensuring accuracy in written methods. Dividing Children struggle to see the grouping structure of division and therefore find the compact and long division methods challenging. ≈ Where there is a remainder that needs to be exchanged, this can cause some children difficulties, although they may be confident when a remainder appears at the end of the calculation.	For addition, see guidance in Stages 2 and 3 about how to develop the formal methods in a conceptual way. Additionally, see the calculation policy for further guidance as well as the NCETM videos for exemplification! https://www.ncetm.org.uk/resources/40532 At this stage, it is advisable to use more abstract models and images e.g. place value counters rather than those that clearly show the values of 10, 100, 1000 etc. If this i needed then it is best to use smaller number (2d or 3d at most) rather than the 4+d recommended in Stage 5. Note the links between the different representations of multiplication and make sure children see these e.g. the grid method can be aligned to the formal written method if you multiply by the largest part first. Use arrays to establish how and why the grid method works and teach the formal method as a contraction of the grid (row by row). As before, try to consistently represent the calculation axb as an array of a columns and b rows to ensure that the scaling concept of multiplication is not lost - you read axb as a multiplied by b (and NOT a lots of b). When carrying out the formal written method, do the grid in parallel so that the exchange and regroup process can take place physically as the number is being transferred column in writing. When teaching division, make reference to the array, which is not just an image for multiplication. There are excellent videos showing how to use the grouping structure with place value counters to support formal division on the NCETM website. Emphasise the two different representations for division - sharing (like dealing out cards and seeing how many cards each person gets) and grouping (pulling up groups of a given size and and seeing how many groups you can create). Younger children tend to favour the sharing structure but you need your children to be comfortable with grouping as this is the basis for long division. Long division is just a less efficient form of compact division (where you calculate the remainder each time MENTALLY as opposed to in writing in long division). It is advised that you focus on compact division as the most efficient method if necessary.

Many children misinterpret remainders in calculations and are unable to explain the impact of a remainder in the context of the question e.g. needing an extra coach to carry the 'remaining' children	
Activities	**Show me what you know**
	Click here to access files in Google drive

Stage 5	Unit 4: Exploring Shape	
Stage 4 support overview	**Stage 5 core learning overview**	**Stage 6 extension overview**
➢ identify lines of symmetry in 2-D shapes presented in different orientations ➢ identify acute and obtuse angles and compare and order angles up to two right angles by size ➢ compare and classify geometric shapes, including quadrilaterals and triangles, based on their properties and sizes	➢ know angles are measured in degrees: estimate and compare acute, obtuse and reflex angles ➢ use the properties of rectangles to deduce related facts and find missing lengths and angles ➢ distinguish between regular and irregular polygons based on reasoning about equal sides and angles.	➢ **illustrate and name parts of circles, including radius, diameter and circumference and know that the diameter is twice the radius** ➢ **… and find unknown angles in any triangles, quadrilaterals, and regular polygons** ➢ **recognise angles where they meet at a point, are on a straight line, or are vertically opposite, and find missing angles** ➢ **compare and classify geometric shapes based on their properties and sizes**

Key learning steps	Key Vocabulary	
1. I can recognise a reflex angle 2. I can estimate angles in degrees (acute, obtuse, reflex) 3. I can compare and order angles (acute, right, obtuse, reflex) 4. I can describe the properties of rectangles 5. I can use the properties of rectangles to find missing lengths and angles 6. I can identify regular polygons and explain my reasoning 7. I can identify irregular polygons and explain my reasoning 8. I can sort regular and irregular polygons using Venn diagrams (3 criteria) and Carroll diagrams (2 criteria)	angle acute right (angle) obtuse reflex degrees compare order estimate greater than less than rectangle equal	opposite parallel symmetry polygon regular irregular properties criterion, criteria Venn diagram Carroll diagram justify explain

Show me... , And another ...	Convince me	What's the same? What's different? (Odd one out)	Always, sometimes, never
... a reflex angle ... an angle of approximately 20	... that a house-shape pentagon is not a regular polygon	scalene, equilateral, right-angled, isosceles	Polygons are symmetrical

degrees .. an angle of approximately 200 degrees ... an angle that is greater than this one ... a rectangle that is half the size of this one ... which of these shapes is regular? ... which of these shapes is a quadrilateral?	... that this angle is approximately 200 degrees	kite, parallelogram, rectangle, square circle, triangle, quadrilateral, pentagon acute, obtuse, reflex, right regular and irregular	Symmetrical shapes are regular There is a 2-sided polygon Reflex angles always have an acute angle on their 'other side' Shapes with all right angles are regular

Misconceptions	**Guidance**
Children sometimes confuse reflex, obtuse and acute angles - they also forget about right angles being between acute and obtuse and that 180 degrees separates obtuse and reflex. When estimating angle sizes, children find it hard to work with a scale centered around 90 and 360 - they cannot quickly find half of 90 or a quarter of it to use their sense of the size in a numeric way. When solving problems using shape properties, children may forget that the symmetry of a shape tells them extra information not shown on the diagram e.g. if you know one side of a rectangle, then you immediately know a second and similarly with angles. Children do not always understand that polygons are the shape family containing ALL closed shapes made of straight sides - they think that triangles and quadrilaterals are different and therefore that polygons 'begin' with pentagons. Some of this is connected to the language so it can be worth remarking that we can call a triangle a trigon - but we don't! Children often interpret the meaning of 'irregular' to be 'completely irregular' i.e. that all the sides and angles are different. They don't see irregular as meaning 'just not regular' and so they do not believe that shapes with 5 equal sides and one different length side are irregular.	Build up familiarity with 90 and 360 as numbers on which angle is centered - make reference to the historical and cultural reasons for this (originally there were thought to be 360 days in a year hence 360 was one full turn/rotation/cycle) Make reference to angles in a shape to help build children's angle sense e.g. an equilateral triangle has 60 degree angles so make comparisons to this When describing reflex angles make sure children are clear on which 'side' of the angle you are looking at - always mark the angle if possible. Use examples of solving problems using shape facts such as finding missing sides and angles using perimeter or symmetry. Explore the different types of polygons and what makes one irregular - get children to find symmetrical but irregular examples. Make the most of practical opportunities for sorting such as children getting themselves onto a Carroll diagram/using sorting hoops on the playground to sort objects and shapes.

Because of the high language demands, pupils may forget shape names of polygons and how they link to number of sides. Children may label Venn or Carroll diagrams incorrectly (or not at all) and particularly may not allow for all possibilities to be shown in a Caroll diagram. When using Venn diagrams, they sometimes forget that numbers can go in the middle and outside.	
Activities	**Show me what you know**
NRICH: Egyptian Rope ** NRICH: Property Chart • NRICH: Shapely Pairs • NRICH: Quadrilaterals Game • BOWLAND assessments: Three of a Kind BOWLAND assessments: Rods and Triangles	Click here to access files in Google drive

Stage 5	Unit 5: Generalising Arithmetic	
Stage 4 support overview	**Stage 5 core learning overview**	**Stage 6 extension overview**
➢ add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate ➢ multiply two-digit and three-digit numbers by a one-digit number using formal written layout ➢ use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers ➢ solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why	➢ add and subtract numbers mentally with increasingly large numbers ➢ add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction) ➢ multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers ➢ divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context ➢ multiply and divide whole numbers and those involving decimals by 10, 100 and 1000 ➢ solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why	➢ **perform mental calculations, including with mixed operations and large numbers** ➢ **multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication** ➢ **divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context** ➢ **multiply one-digit numbers with up to two decimal places by whole numbers** ➢ **use written division methods in cases where the answer has up to two decimal places** ➢ **solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why** ➢ **solve problems involving addition, subtraction, multiplication and division**
Key learning steps		**Key Vocabulary**

1. I can add and subtract numbers involving three and four digits mentally 2. I can add and subtract two three/four digit numbers using a column method 3. I can multiply a four digit number by a one digit number using a written method 4. I can multiply a four digit number by a two digit number using a written method of long multiplication 5. I can divide a four digit number by a one digit number and find remainders using a written method of short division 6. I can multiply whole numbers and decimals by 10, 100 and 1000 7. I can divide whole numbers and decimals by 10, 100 and 1000 8. I can solve multi step addition and subtraction problems choosing the correct operation and using the most appropriate methods	ones tens hundreds thousands tenths hundredths mental mentally add sum of total subtract take away minus less than more than multiply multiple divide	round rounding product calculate digit column addition column subtraction carry borrow inverse operation check place value partition remainders lots of groups of nearest decimal

Show me... , And another ...	Convince me	What's the same? What's different? (Odd one out)	Always, sometimes, never
...an addition calculation with answer 4765 ...how you multiply 4523 x 6 using the grid method? using partitioning? using a column method? ... how you divide 5683 ÷ 4 using place value counters? using a written method? ... 45 x 100 ...4.5 x 100 ... 45 ÷ 10 ... the calculation you would do to find the missing numbers: 6.5–9.8=	... that 12.3 + 9.8 IS NOT EQUAL to 21.11 ... if 0.4×7=2.8, then 2.4×7=16.8 ... that there are only two numbers with a sum of 35 and a difference of 7.	4005 - 1997; 4004 - 1996; 4005 + 1997; 4004 + 1998 1234 x 5; 123.4 x 5; 1234 x 10; 123.4 x 10	When you multiply a number by 100, you just add two zeroes on the end When you add two numbers together, if you increase them both by 1 the answer will be 2 higher. When you subtract two numbers, if you increase them both by 1, the answer will be the same.

4.8 ÷= 0.96 1⁄8 of=40 Choose a number to put into a calculator. Add 472 (or multiply by 26,			

Misconceptions	Guidance
Children can sometimes be found failing to carry or borrow when they need to as well as carrying or borrowing when it is not necessary Once numbers have been carried, some children will forget to include the tens or hundreds they have carried In subtraction, children will sometimes borrow from the wrong column. Additionally, they may place the smallest number at the top of the calculation when using column subtraction. Children sometimes use the incorrect operation when checking In solving problems, children may complete the first step of a problem and use it as the answer without going any further/ Children sometimes add instead of multiplying e.g. they may add on 10 100 or 1000 when multiplying by 10 100 1000 In long multiplication, children sometimes miss out one of the parts of the calculation In division, children get confused when there is a remainder within the calculation and may forget to use it or may put the remainder itself as the answer.	Ensure children understand and are secure with column addition and subtraction before attempting written methods of multiplication and division. Similarly they need to be able to complete multiplication by 10, 100, 1000 prior to using the grid really as this assumes place value knowledge of this type. Similarly, strong recall of times tables is required to be proficient with this level of multiplication and division to ensure this is solid to allow time and focus on the concepts themselves. Encourage children to use their estimate when calculating so that they can gain a sense of whether their answer is correct. Use the progression from grid method to long multiplication column method shown in the calculation policy to unpick why this method works. Use place value counters to teach division methods to show how the remainders work - see the NCETM video for more guidance. Have children explain multiplying and dividing by 10, 100 and 1000 using place value counters to see why each ten becomes a hundred and hence the digit from the tens column ends up in the hundreds column. It is important to avoid simply 'adding a 0' and the like because this falls down when decimals are involved. Draw attention to language used within addition and subtraction problems - what clues do we have that we are being asked to add or subtract? Which numbers are involved? Make use of the bar model as a tool for representing the problem in a visual context - this will make it easier for children to decide what operations they

	need to carry out. Make children aware that some problems involve 2 steps or more which could mean 2 or more different operations.
Activities	**Show me what you know**
NRICH: Twenty Divided Into Six ** NRICH: Reach 100 *** NRICH: Two and Two *** NRICH: Journeys in Numberland *	Click here to access files in Google drive

Stage 5	Unit 6: Reasoning with Measures	
Stage 4 support overview	**Stage 5 core learning overview**	**Stage 6 extension overview**
➢ estimate, compare and calculate different measures, including money in pounds and pence ➢ measure and calculate the perimeter of a rectilinear figure (including squares) in centimetres and metres ➢ find the area of rectilinear shapes by counting squares	➢ measure and calculate the perimeter of composite rectilinear shapes in centimetres and metres ➢ calculate and compare the area of rectangles (including squares), and including using standard units, square centimetres (cm^2) and square metres(m^2) and estimate the area of irregular shapes ➢ "estimate volume [for example, using ➢ 1 cm^3 blocks to build cuboids (including cubes)] and capacity [for example, using water]"	➢ **recognise that shapes with the same areas can have different perimeters and vice versa** ➢ **calculate the area of parallelograms and triangles** ➢ **recognise when it is possible to use formulae for area of shapes** ➢ **calculate, estimate and compare volume of cubes and cuboids using standard units, including cubic centimetres (cm^3) and cubic metres (m^3), and extending to other units [for example, mm^3 and km^3]** ➢ **recognise when it is possible to use formulae for volume of shapes**
Key learning steps		**Key Vocabulary**
1. I can calculate the perimeter of a composite shape (eg L, E, F shapes) by adding up side lengths, including cases where not all sides are given directly 2. "I can use a ruler and pair of compasses to draw a triangle with given sides. 3. I can use a set square and ruler, or squared paper, to draw a rectangle of a given size" 4. I can calculate and compare the area of rectangles and squares by multiplying the length by the width and giving the answer in the correct units (mm2, cm2, m2) 5. I can calculate and compare the area of composite shapes by splitting them into rectangles, giving the answer in the correct units (mm2, cm2, m2) 6. I can estimate the area of irregular shapes by counting squares and part squares 7. I can explain that the volume of a cuboid is measured by counting how many unit cubes it takes to fill it (eg with multilink or 1cm cubes) 8. I can estimate and compare the volume of small boxes in cm3 by counting how many (closely packed) 1cm cubes it takes to fill them 9. I can estimate the capacity of different containers using liquids and measuring jugs. I know that capacity in ml is the same as volume in cm3, and that 1 litre is the capacity of a 10 x 10 x 10cm cube		capacity; cm2; cm3; column; container; cube; cuboid; explain; length; litre; m2; ml; mm2; multilink; row; volume; width

Show me... , And another ...	Convince me	What's the same? What's different? (Odd one out)	Always, sometimes, never
How to draw an accurate equilateral triangle.	That the area of a 3cm by 2cm rectangle is 600mm2.	A 4 x 9 rectangle, a 6 x 6 square, a 3 x 12 rectangle and a 5 x 7 rectangle	The taller glass holds more drink. A cube-shaped box with (internal) sides of 10cm will hold a litre of water.

Misconceptions	Guidance
Only adding given lengths in perimeter; inability to deduce missing lengths; confusing area with perimeter; difficulty in using compasses (possibly bad-design); inability to dissect reliably; confusing area and volume; lack of concept of volume; thinking multilink cubes are 1cm3 (they are 8cm3 each); not understanding relationship of capacity to volume; failure to state units when measuring Children sometimes add instead of multiplying e.g. they may add on 10 100 or 1000 when multiplying by 10 100 1000 In long multiplication, children sometimes miss out one of the parts of the calculation In division, children get confused when there is a remainder within the calculation and may forget to use it or may put the remainder itself as the answer.	Some children will need to be taught to deduce missing lengths on a diagram; others will find it obvious. The key is to realise why it is necessary: keep emphasising the 'journey round the shape' - how long is this (possibly unmarked) piece? The drawing work is vital preparation for future work. Practical experience of using compasses is invaluable, but so is understanding of why they are needed to draw a triangle. Area and perimeter work now allows A = l x w but is now extended to composite shapes, including areas of shapes with 'holes'. There is more emphasis on correct units, but a return to counting squares for 'irregular' shapes - eg on a geoboard. Work on volume aims to establish the principle of counting cubes - eg in multilink solids initially (since 1cm cubes are fiddly). NO FORMULAS! Filling a box with cm cubes links volume to capacity. Children will need experience with different shape containers: some will still think a tall thin glass holds more than a short fat one. Measuring cylinders could be used for comparison.

Activities	Show me what you know
NRICH: Area and Perimeter * NRICH: Numerically Equal ** NRICH: Shaping It * NRICH: Cubes * NRICH: Fitted *** NRICH: Brush Loads * NRICH: Making Boxes **	Click here to access files in Google drive

Stage 5	Unit 7: Discovering Equivalence	
Stage 4 support overview	**Stage 5 core learning overview**	**Stage 6 extension overview**
➢ recognise and show, using diagrams, families of common equivalent fractions ➢ count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10 ➢ recognise and write decimal equivalents of any number of tenths or hundredths ➢ recognise and write decimal equivalents to 1/4,1/2, 3/4	➢ recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number [for example, 2/5 + 4/5 = 6/5 = 1 1/5 ➢ compare and order fractions whose denominators are all multiples of the same number ➢ identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths ➢ count up and down in hundredths; recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten ➢ read and write decimal numbers as fractions [for example, 0.71 = 71/100] ➢ recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents ➢ recognise the per cent symbol (%) and understand that per cent relates to 'number of parts per hundred' ➢ write percentages as a fraction with denominator 100, and as a decimal ➢ solve problems which require knowing percentage and decimal equivalents of 1/2, 1/4, 1/5, 2/5, 4/5 and those fractions with a denominator of a multiple of 10 or 25.	➢ **compare and order fractions, including fractions > 1** ➢ **use common factors to simplify fractions; use common multiples to express fractions in the same denomination** ➢ **solve problems involving the calculation of percentages [for example, of measures, and such as 15% of 360] and the use of percentages for comparison** ➢ **associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction [for example, 3/8]** ➢ **recall and use equivalences between simple fractions, decimals and percentages, including in different contexts**
Key learning steps		**Key Vocabulary**

1. I can recognise mixed numbers and improper fractions and convert between the two 2. I can compare and order fractions by finding a common denominator 3. I can find equivalent fractions by multiplying or dividing the numerator and denominator by the same number 4. I can count up and down in hundredths and thousandths and can find a hundredth by dividing a number by 100 5. I can read and write decimals as fractions by looking at the lowest place value heading (e.g. 0.71 has 71 hundredths so is 71/100) 6. I can explain that % = number of parts per hundred 7. I can write percentages as fractions, showing numbers out of a hundred, and can convert these to decimals by dividing the numerator by the denominator (dividing by 100) 8. I can convert 1/2, 1/4, 1/5, 2/5, 4/5 into decimals and percentages and use these to solve problems	fraction mixed number improper fraction convert equivalent decimal hundredth thousandth percentage parts per hundred compare order denominator numerator common denominator place value

Show me... , And another ...	Convince me	What's the same? What's different? (Odd one out)	Always, sometimes, never
... fraction equivalent to (i) 3/4,(ii) 7/10 ... a mixed number, and another, … ... a proper fraction, and another, … ... an improper fraction, and another, … ... as many different representations of 5/4 as you can (use symbols, writing, images and models) ... how you order: 3/10, 3/4, 1/5, 3/20	... that ten thousandths is equivalent to one hundredth ... that 7/12 < 2/3 ... that 0.3 = 30/100 ... that 1/2 cannot be written as 1.2 ... 13/10 = 1 3/10 ... 0.1 = 10%	3/10, 8/3, 3 1/10, 25/100 0.1, 3/10, 0.25, 1/4 0.2, 20%, 2/10, 2.1	... percentages are fractions with a denominator of 100 ... every percentage can be written as a fraction ... every fraction can be written as a percentage ... improper fractions must be greater than 1 ... mixed numbers are better than improper fractions

Misconceptions	Guidance
"Many children believe that you cannot have a fraction where the numerator is greater than the denominator and they may reattempt the question or alternatively turn their fraction upside down to avoid giving an answer in this	"The exercises in stage 4 of counting in tenths and now in hundredths is key in exposing children to the idea that you can have a fraction with a numerator that is greater than the denominator e.g. 11 tenths. If this counting has been

form. When finding an equivalent fraction, some children will do different things to the numerator and denominator. Children may confuse 'hundreds' and 'hundredths' or 'thousands' and 'thousandths' Children may experience some confusion over place value headings after the decimal point - make sure these are consistent with your headings before the decimal point e.g. if you are using 1000s, 100s, 10s, 1s then you need to use 1/10s, 1/100s etc Occasionally children may include more than one decimal point. As in Stage 4, some children believe that adding a zero = multiplying by 10 (e.g. 0.1 x 10 = 0.10 instead of moving the digits up a space"	accompanied by images to support the process this will be even stronger. Use visual representations to explore the connection between improper fractions and mixed numbers e.g. 11/10 as one whole and 1/10. Gradually move towards using division as a process to convert an improper fraction e.g. 7/3 means 7 divided by 3 which is 2 remainder 1 or 2 and 1/3. The step based on comparing and ordering fractions where the denominators are multiples is key to good fraction addition and subtraction later - therefore spend time here ensuring that children are confident to 'translate' fractions into the same language before ordering. Make use of models and images to justify their equivalent representations. When introducing percentages, the 100-square is a particularly useful image and will also help to reinforce the equivalence of 10 hundredths and 1 tenth (plus the other multiples of ten). Try to constantly relate the symbol % with /100. Try not to get children to believe that 24% = 0.24 x 100 (which is not true!) and instead encourage them to believe that 24% = 0.24 itself. It is useful, particularly for more able children, to relate the initial work on mixed numbers and improper fractions to percentages that are greater than 100%. The final small step requires children to just 'know' the equivalences for common fractions - focus on speed recall here."
Activities	**Show me what you know**
	Click here to access files in Google drive

Stage 5	Unit 8: Investigating Statistics	
Stage 4 support overview	**Stage 5 core learning overview**	**Stage 6 extension overview**
➢ interpret and present discrete and continuous data using appropriate graphical methods, including bar charts and time graphs ➢ solve comparison, sum and difference problems using information presented in bar charts, pictograms, tables and other graphs	➢ solve comparison, sum and difference problems using information presented in a line graph ➢ complete, read and interpret information in tables, including timetables	➢ **interpret and construct pie charts and line graphs and use these to solve problems** ➢ **calculate and interpret the mean as an average**

Key learning steps	Key Vocabulary	
1. I can find a sum or a difference to answer a question about a line graph e.g. how many pieces of data were greater than 11 2. I can make a comparison between two data points on a line graph e.g. how many more were sold on Wed than on Mon? 3. I can read a timetable to find a journey start time or end time. 4. I can read a timetable to find a journey duration 5. I can place information in the correct place in a table. 6. I can find a given piece of information from a table.	sum total altogether difference how many more.../fewer... line graph	table row column heading information timetable start time end time duration

Show me... , And another ...	Convince me	What's the same? What's different? (Odd one out)	Always, sometimes, never
... how you would find the temperature in May from this graph? ... how you would find the difference between the temperature in June and in January using this graph? ... how you would estimate the temperature in between Sep and Oct using this graph ... a bus on this timetable that leaves	... there are two ways to find out how many results were greater than 40 from this graph ... this bus is quicker than the one at 1805	sum, total, altogether, more, difference, how many fewer, how many more	...timetables are always read vertically ... line graphs are more useful than bar charts because they tell you values in between your data

before 7am ... what time the 0815 bus gets to Crewe ... the last train I can catch to get back home to Torquay by 1800			

Misconceptions	Guidance
Children struggle to work out which pieces of information they need to read off a graph e.g. if the question says "how many scored fewer than 11" they do not realise they need all the frequencies of the values up to 10 With timetables, children sometimes struggle to read vertically and to realise that all the times in a single column represent the same bus or train etc. Issues with 24hr clock and with time generally may also appear here. When a timetable has a blank or a dashed line to show a train or bus doesn't stop at a destination, this can confuse children.	Give children lots of different graphs to look at with questions that progress from simply retrieving a single piece of information to those requiring the collection and addition/subtraction of multiple pieces of information. Make use of real timetables for trains, buses and so on when looking at timetables - try to have some where a single bus has a whole column to itself and somewhere multiple trains are listed in the same column. This is a good opportunity to revisit number bonds to 60 to find journey durations as well as to look at the 24 hour clock.

Activities	Show me what you know
ITP Line Graphs http://mathsframe.co.uk/en/resources/resource/111/itp_line_graph	Click here to access files in Google drive

Stage 5	Unit 9:Solving Number Problems	
Stage 4 support overview	**Stage 5 core learning overview**	**Stage 6 extension overview**
➢ find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths ➢ "use place value, known and derived facts to multiply and divide mentally, including: ➢ multiplying by 0 and 1; ➢ dividing by 1; ➢ multiplying together three numbers" ➢ multiply two-digit and three-digit numbers by a one-digit number using formal written layout ➢ solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects ➢ solve simple measure and money problems involving fractions and decimals to two decimal places	➢ multiply and divide whole numbers and those involving decimals by 10, 100 and 1000 ➢ multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers ➢ divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context ➢ solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes ➢ solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign ➢ solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates ➢ solve problems involving number up to three decimal places	➢ **multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places** ➢ **perform mental calculations, including with mixed operations and large numbers** ➢ **multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication** ➢ **divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context** ➢ **multiply one-digit numbers with up to two decimal places by whole numbers** ➢ **use written division methods in cases where the answer has up to two decimal places** ➢ **solve problems involving addition, subtraction, multiplication and division** ➢ **use their knowledge of the order of operations to carry out calculations involving the four operations** ➢ **solve problems which require answers to be rounded to specified degrees of accuracy** ➢ **express missing number problems algebraically** ➢ **find pairs of numbers that satisfy an**

		➢ **equation with two unknowns** ➢ **enumerate possibilities of combinations of two variables**

Key learning steps	**Key Vocabulary**	
1. I can multiply and divide whole numbers by 10, 100, 1000 2. I can multiply and divide decimals by 10, 100, 1000 3. I can use my knowledge of factors, multiples, squares and cubes to solve multiplication and division problems 4. I can use efficient written methods to solve addition, subtraction, multiplication and division problems 5. I can find missing numbers in problems involving the = sign. 6. I can solve fractional scaling problems. 7. I can solve problems involving simple rates. 8. I can work with problems involving numbers with up to 3 decimal places	multiply divide add subtract place value - including tenths, hundredths, thousandths and decimal places remainder factor	t multiple square(d) cube(d) equals scale scale up by/scale down by rate per

Show me... , And another ...	**Convince me**	**What's the same? What's different? (Odd one out)**	**Always, sometimes, never**
... the number that is 1000 times bigger than 12? 1.3? 4.02 ... the number that is 1000 times smaller than 14000, 120, 14 ... what the missing number must be: 14.3 x 100 = ☐ 2 x ☐ + 11 = 35 7 = 2 + ☐ ÷ 6 ... how you would represent this problem: Sharon and Tim each had a collection of football stickers. Tim had 5 times as many as Sharon. He had 150. How many did they have altogether?	... that 230 ÷ 1000 = 0.23 ... that 4.5 x 1000 = 4500 ... that 72 has an even number of factors ... how you would calculate 15 squared? 6 cubed?	46 × 10, 460 ÷ 100, 46 × 1000 and 4600 ÷ 1000 98 ÷6, 48 ÷6, 18 ÷6, 78 ÷6 division, finding a fraction of, scaling down	... it is impossible to find all the multiples of 12 because there are an infinite number... ... per means divide ... numbers have an even number of factors

Misconceptions	Guidance
As in Stage 4, children may struggle with multiplying and dividing by 10, 100, 1000 due to a reliance on a short cut of adding or removing zeros. There may be confusion over the meanings of and differences between squared and cubed as well as a failure to see how these number facts (including factors and multiples) relate to multiplication and division. Sometimes, children may struggle when a division problem has a remainder to know how to interpret this or how to represent it. When solving problems children typically encounter the following issues: - difficulty in pulling out the key information from any text - difficulty deciding which calculation(s) to carry out and, where relevant, in what order (particularly where the language is not of their preferred version e.g. scaling rather than lots of - difficulty interpreting their answer in the context of the problem. Children may find scaling problems challenging if they do not naturally represent thee as multiplication. Children may struggle with the idea that a rate is a division and use of the word per.	Note: this unit does not directly cover the greyed out objectives (which have been studied already) - however, if your children are not secure on these, you need to address this before moving them on to the more complex skills contained in this unit. Mult/Div by 10, 100, 1000 - as in Stage 4, there is a need to teach this without immediate resort to the shortcut re adding or removing zeroes. Instead, try the unitisation approach which relies on the connections between place value headings. For example, 1.23 x 10 is 1 one, 2 tenths and 3 hundredths that all need to be SCALED up by 10 giving 1 ten, 2 ones and 3 tenths or 12.3 in shorthand. You will need place value heading practically as a prompt for this. Factors, Multiplies, Squares and Cubes This is a chance to revisit these concepts from Unit 1 - Pattern Sniffing. Children do not always see these as a form of multiplication and division so explore these fully to apply the skills of calculation. E.g. Sally was asked to find all the factors of 48. She found 8. These were, 1, 48, 2, 24, 3, 16, 4, 12. Did she find them all? How do you know? Solving problems Give children lots of opportunities to select the calculations they will do AND to carry them out formally. Make use of the bar model to represent the problems given to help choose the calcs and the order of these. See more guidance on this here at the NCETM https://www.ncetm.org.uk/resources/44565 You may also want to refer to the AET Calc policy to support you in dealing with any issues in the calculations themselves. Make sure that some problems encountered are more abstract (i.e. NOT based on real life situations) so that children are exposed to some of the technical language also. These can still be represented with the bar model but may need more unpicking. E.g. Ella is thinking of two numbers with a sum of 12. The second number is twice as large as the first one. What are they? Also consider problems where children must 'balance' a calculation containing an = sign e.g. 14 x 5 = 85 - ?. Again this can be represented by a bar and this is an important precursor the algebra work of Stage 7 that will be needed to

	secure a good GCSE. Rates This will be a new concept to the children at this point so introduce it carefully with simple examples making use of the language of 'per' to identify this type of question. E.g. rates of hire of equipment or speeds of different transport. Encourage children to see these problems as examples of SCALING e.g. if it costs £20 per hour to hire a canoe, then you can use scaling to find out the costs of hiring for 6 hours. This is another example of a problem that can be represented well using the bar model to help the child 'see' what they need to do in terms of calculations.
Activities	**Show me what you know**
	Click here to access files in Google drive